Lesson 7: Inequivalent Equations

- Let's see what happens when we square each side of an equation.

7.1: 2 and -2

What do you notice? What do you wonder?

- $x^2 = 4$
- $x^2 = \text{-}4$
- $(x-2)(x+2) = 0$
- $x = \sqrt{4}$

7.2: Careful When You Take the Square Root

Tyler was solving this equation:

$$x^2 - 1 = 3$$

He said, "I can add 1 to each side of the equation and it doesn't change the equation. I get $x^2 = 4$."

1. Priya said, “It does change the equation. It just doesn’t change the solutions!” Then she showed these two graphs.

Figure A

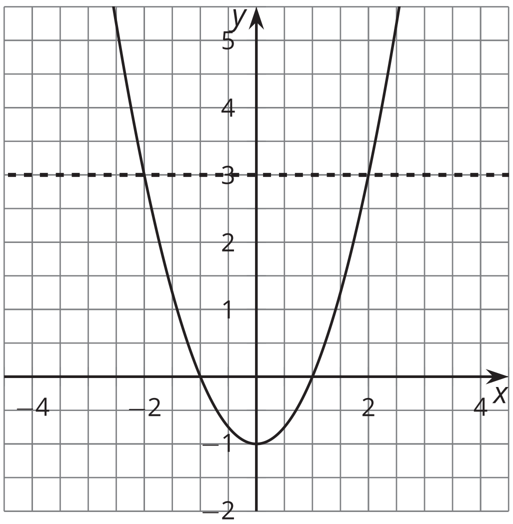
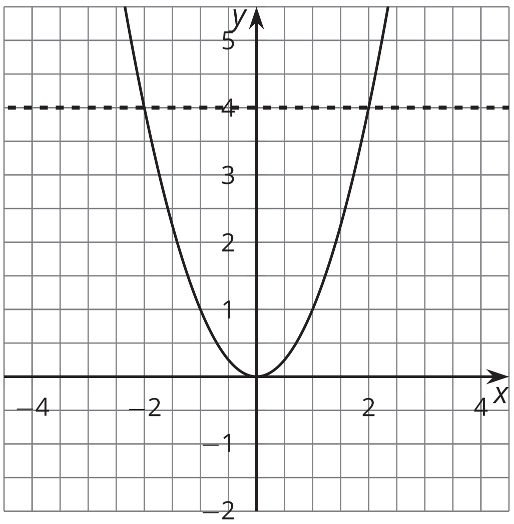


Figure B



a. How can you see the solution to the equation $x^2 - 1 = 3$ in Figure A?

b. How can you see the solution to the equation $x^2 = 4$ in Figure B?

c. Use the graphs to explain why the equations have the same solutions.

2. Tyler said, “Now I can take the square root of each side to get the solution to $x^2 = 4$. The square root of x^2 is x. The square root of 4 is 2.” He wrote:

$$x^2 = 4$$
$$\sqrt{x^2} = \sqrt{4}$$
$$x = 2$$

Priya said, “But the graphs show that there are *two* solutions!” What went wrong?

7.3: Another Way to Solve

Han was solving this equation:

$$\frac{x+3}{2}=4$$

He said, "I know that half of $x+3$ is 4. So $x+3$ must be 8, since half of 8 is 4. This means that x is 5."

1. Use Han's reasoning to solve this equation: $(x+3)^2=4$.

2. What advice would you give to someone who was going to solve an equation like $(x+3)^2=4$?

7.4: What Happens When You Square Each Side?

Mai was solving this equation:

$$\sqrt{x-1} = 3$$

She said, "I can square each side of the equation to get another equation with the same solutions." Then she wrote:

$$\begin{aligned} \sqrt{x-1} &= 3 \\ (\sqrt{x-1})^2 &= 3^2 \\ x-1 &= 9 \\ x &= 10 \end{aligned}$$

1. Check to see if her solution makes the original equation true.

2. Andre said, "I tried your technique to solve

$$\sqrt{x-1} = \text{-}3$$

but it didn't work." Why doesn't it work? Explain or show your reasoning.

7.5: Solve These Equations With Square Roots in Them

Find the solution(s) to each of these equations, or explain why there is no solution.

1. $\sqrt{t+4} = 3$

2. $-10 = -\sqrt{a}$

3. $\sqrt{3-w} - 4 = 0$

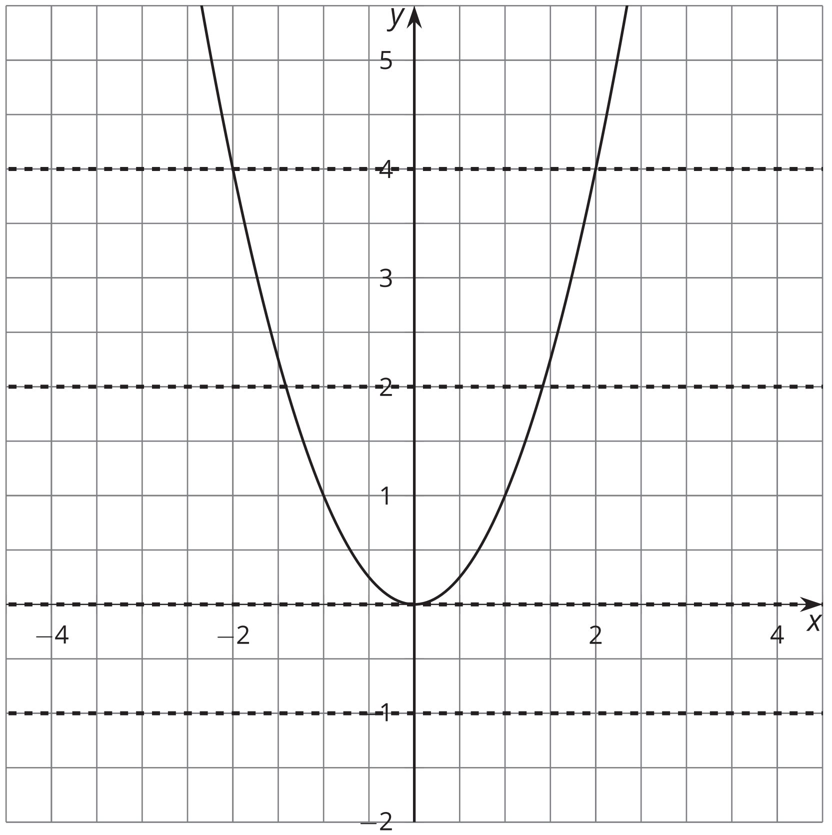
4. $\sqrt{z} + 9 = 0$

Are you ready for more?

Are there values of a and b so that the equation $\sqrt{t+a} = b$ has more than one solution? Explain your reasoning.

Lesson 7 Summary

Every positive number has *two* square roots. You can see this by looking at the graph of $y = x^2$:



If y is a positive number like 4, then we can see that $y = 4$ crosses the graph in two places, so the equation $x^2 = 4$ will have two solutions, namely, $\sqrt{4}$ and -$\sqrt{4}$. This is true for any positive number a: $y = a$ will cross the graph in two places, and $x^2 = a$ will have two solutions, $x = \sqrt{a}$ and $x = -\sqrt{a}$.

When we have a square root in an equation like $\sqrt{t} - 6 = 0$, we can isolate the square root and then square each side:

$$\begin{aligned} \sqrt{t} - 6 &= 0 \\ \sqrt{t} &= 6 \\ t &= 6^2 \\ t &= 36 \end{aligned}$$

But sometimes, squaring each side of an equation gives results that aren't solutions to the original equation. For example:

$$\begin{aligned} \sqrt{t} + 6 &= 0 \\ \sqrt{t} &= \text{-}6 \\ t &= (\text{-}6)^2 \\ t &= 36 \end{aligned}$$

Note that 36 is *not* a solution to the original equation, because $\sqrt{36} + 6$ doesn't equal 0. In fact, $\sqrt{t} + 6 = 0$ has no solutions, because it's impossible for the sum of two positive numbers to be zero.

Remember: sometimes the new equation has solutions that the old equation doesn't have. Always check your solutions in the original equation!