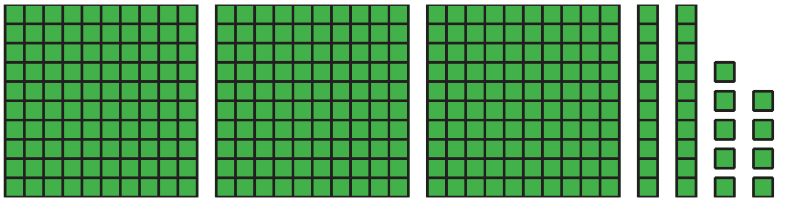
Lesson 2: Funding the Future

- Let's look at some other things that polynomials can model.

2.1: Notice and Wonder: Writing Numbers

What do you notice? What do you wonder?



$$300 + 20 + 9$$

3 100s, 2 10s, 9 1s

$$3(10^2) + 2(10^1) + 9(10^0)$$

2.2: Polynomials in the Integers

Consider the polynomial function p given by $p(x) = 5x^3 + 6x^2 + 4x$.

1. Evaluate the function at $x = -5$ and $x = 15$.

2. How does knowing that $5{,}000 + 600 + 40 = 5{,}640$ help you solve the equation $5x^3 + 6x^2 + 4x = 5{,}640$?

Are you ready for more?

Han notices:

- $11^2 = 121$ and $(x+1)^2 = x^2 + 2x + 1$
- $11^3 = 1331$ while $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

The digits in the powers of 11 correspond to the coefficients of the polynomials.

1. Is this still true for 11^4 and $(x+1)^4$? What about 11^5 and $(x+1)^5$?

2. Give a mathematical justification of Han's observation.

2.3: A Yearly Gift

At the end of 12th grade, Clare’s aunt started investing money for her to use after graduating from college four years later. The first deposit was $300. If r is the annual interest rate of the account, then at the end of each school year the balance in the account is multiplied by a growth factor of $x = 1 + r$.

1. After one year, the total value is $300x$. After two years, the total value is $300x \cdot x = 300x^2$. Write an expression for the total value after graduation in terms of x.

2. If Clare’s aunt had invested another $500 at the end of her freshman year, what would the expression be for the total value after graduation in terms of x?

   Pause here for a whole-class discussion.

3. Suppose that $250 was invested at the end of sophomore year, and $400 at the end of junior year in addition to the original $300 and the $500 invested at the end of freshman year. Write an expression for the total value after graduation in terms of x.

4. The total amount y, in dollars, after four years is a function $y = C(x)$ of the growth factor x. If the total Clare receives after graduation is $C(x) = 1{,}580$, use a graph to find the interest rate that the account earned.

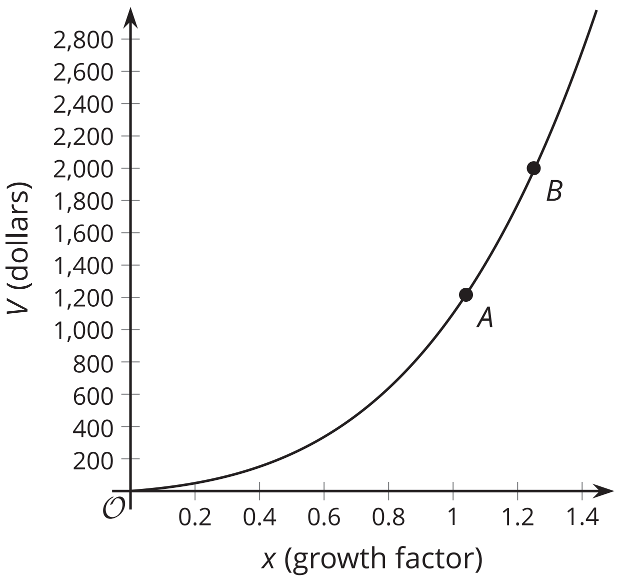
Lesson 2 Summary

Let's say we're going to invest \$200 at an annual interest rate of r. This means at the end of a year, the balance in the account is multiplied by a growth factor of $x = 1 + r$. After the first year, the amount in the account can be expressed as $200x$, which is a polynomial. Similarly, after the second year, the amount will be $200x^2$, after three years, the amount will be $200x^3$, etc.

If an additional \$350 is invested at the end of the first year, we can revise the polynomial. The amount of money in the account after 1 year is the same, but now the amount of money after two years is $(200x + 350)x = 200x^2 + 350x$.

What will the polynomial expression look like if \$400 more is invested at the end of the second year and \$150 more is invested at the end of the third year?
$200x^4 + 350x^3 + 400x^2 + 150x$.

Let $D(x)$ be the amount of money in dollars in the account after four years and x be the growth factor where $D(x) = 200x^4 + 350x^3 + 400x^2 + 150x$. A graph of $y = D(x)$ helps us visualize how the amount in the account after four years depends on different values of x.



We can use this polynomial model to examine the effect of different annual interest rates, or to estimate what the annual interest rate needs to be to achieve a specific quantity at the end of the four years. For example, point A is at $(1.04, D(1.04)) \approx (1.04, 1216)$. From this, we know that the amount in the account after 4 years with an interest rate of 4% each year is approximately \$1,216. Similarly, if we want the account to have \$2,000 after four years, that corresponds to point B, and at that point the growth rate is approximately 1.25 each year, since $(1.25, D(1.25)) \approx (1.25, 2000)$. So an interest rate of 25% will get us there, though we are not likely to find a bank that would offer that rate. Note also that the values $x < 1$ correspond to negative rates, which are also unlikely!

Polynomial models are adaptable to a variety of situations even as they grow in complexity.