Lesson 5: Circumference and Wheels

Let's explore how far different wheels roll.

5.1: A Rope and a Wheel

Han says that you can wrap a 5-foot rope around a wheel with a 2-foot diameter because $\frac{5}{2}$ is less than pi. Do you agree with Han? Explain your reasoning.

5.2: Rolling, Rolling, Rolling

Your teacher will give you a circular object.

1. Follow these instructions to create the drawing:
    - On a separate piece of paper, use a ruler to draw a line all the way across the page.
    - Roll your object along the line and mark where it completes one rotation.
    - Use your object to draw tick marks along the line that are spaced as far apart as the diameter of your object.
2. What do you notice?

3. Use your ruler to measure each distance. Record these values in the first row of the table:

   a. the diameter of your object

   b. how far your object rolled in one complete rotation

   c. the quotient of how far your object rolled divided by the diameter of your object

object	diameter	distance traveled in one rotation	distance ÷ diameter

4. If you wanted to trace two complete rotations of your object, how long of a line would you need?

5. Share your results with your group and record their measurements in the table.

6. If each person in your group rolled their object along the entire length of the classroom, which object would complete the most rotations? Explain or show your reasoning.

5.3: Rotations and Distance

1. A car wheel has a diameter of 20.8 inches.

    a. About how far does the car wheel travel in 1 rotation? 5 rotations? 30 rotations?

    b. Write an equation relating the distance the car travels in inches, c, to the number of wheel rotations, x.

    c. About how many rotations does the car wheel make when the car travels 1 mile? Explain or show your reasoning.

2. A bike wheel has a radius of 13 inches.

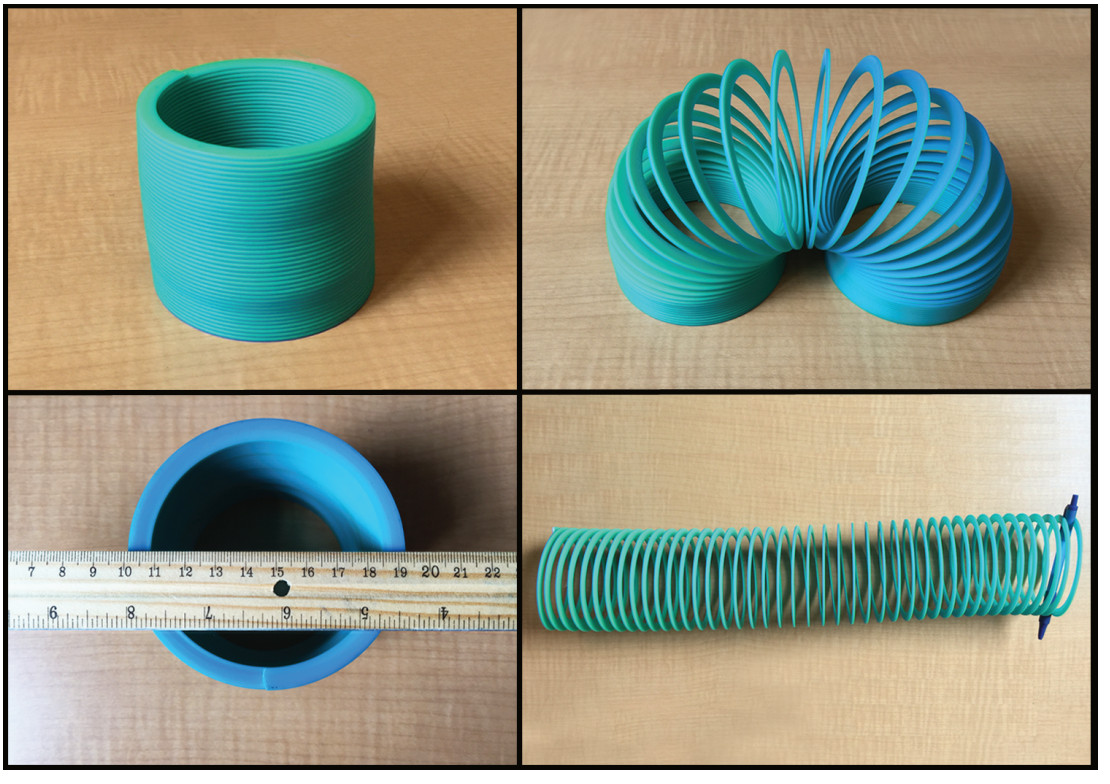
    a. About how far does the bike wheel travel in 1 rotation? 5 rotations? 30 rotations?

    b. Write an equation relating the distance the bike travels in inches, b, to the number of wheel rotations, x.

    c. About how many rotations does the bike wheel make when the bike travels 1 mile? Explain or show your reasoning.

Are you ready for more?

Here are some photos of a spring toy.



If you could stretch out the spring completely straight, how long would it be? Explain or show your reasoning.

5.4: Rotations and Speed

The circumference of a car wheel is about 65 inches.

1. If the car wheel rotates once per second, how far does the car travel in one minute?

2. If the car wheel rotates once per second, about how many miles does the car travel in one hour?

3. If the car wheel rotates 5 times per second, about how many miles does the car travel in one hour?

4. If the car is traveling 65 miles per hour, about how many times per second does the wheel rotate?

Lesson 5 Summary

The circumference of a circle is the distance around the circle. This is also how far the circle rolls on flat ground in one rotation. For example, a bicycle wheel with a diameter of 24 inches has a circumference of 24π inches and will roll 24π inches (or 2π feet) in one complete rotation. There is an equation relating the number of rotations of the wheel to the distance it has traveled. To see why, let's look at a table showing how far the bike travels when the wheel makes different numbers of rotations.

number of rotations	distance traveled (feet)
1	2π
2	4π
3	6π
10	20π
50	100π
x	?

In the table, we see that the relationship between the distance traveled and the number of wheel rotations is a proportional relationship. The constant of proportionality is 2π.

To find the missing value in the last row of the table, note that each rotation of the wheel contributes 2π feet of distance traveled. So after x rotations the bike will travel $2\pi x$ feet. If d is the distance, in feet, traveled when this wheel makes x rotations, we have the relationship:

$$d = 2\pi x$$