Lesson 19: Beyond Circles

- Let's use trigonometric functions to model data.

19.1: Notice and Wonder: Examining Data

Here is some data that we will study in today's lesson.

day	amount
1	0.99
2	1.00
3	0.98
4	0.93
5	0.86
6	0.77
7	0.67
8	0.57
9	0.46
10	0.37

day	amount
11	0.28
12	0.19
13	0.13
14	0.07
15	0.03
16	0.01
17	0.00
18	0.01
19	0.04
20	0.09

day	amount
21	0.16
22	0.24
23	0.33
24	0.43
25	0.54
26	0.65
27	0.76
28	0.85
29	0.92
30	0.98
31	1.00

What do you notice? What do you wonder?

19.2: Watching the Evening Sky

The data from the warm-up is the amount of the Moon that is visible from a particular location on Earth at midnight for each day in January 2018. A value of 1 represents a full moon in which all of illuminated portion of the moon's face is visible. A value of 0.25 means one fourth of the illuminated portion of the moon's face is visible.

1. What is an appropriate midline for modeling the Moon data? What about the amplitude? Explain your reasoning.

2. What is an appropriate period for modeling the Moon data? Explain your reasoning.

3. Choose a sine or cosine function to model the data. What is the horizontal translation for your choice of function?

4. Propose a function to model the Moon data. Explain the meaning of each parameter in your model and specify units for the input and output of your function.

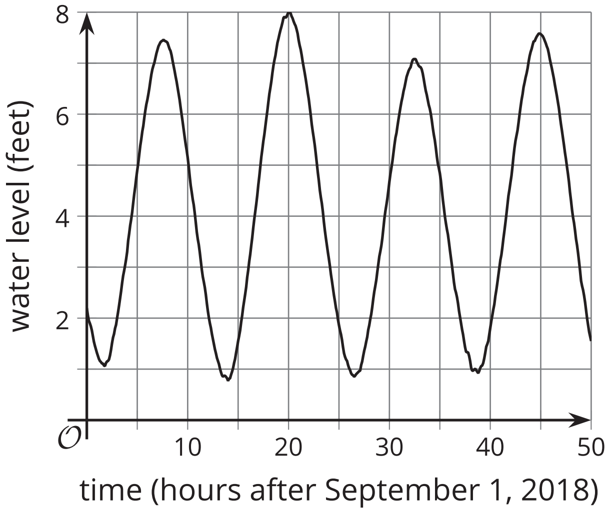
5. Plot the data using graphing technology and check your choice of parameters (midline, amplitude, period, horizontal translation). What changes did you make to your model?

6. Use your model to predict when the next two full moons will be in 2018. Are your predictions accurate?

7. How much of the Moon do you expect to be visible on your birthday? Explain your reasoning.

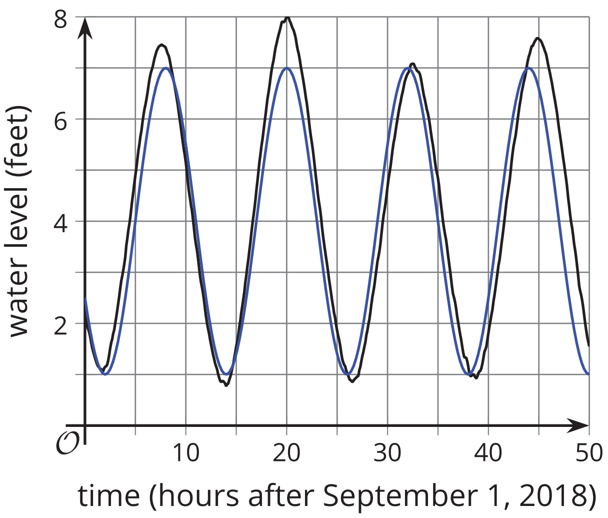
Lesson 19 Summary

Sometimes a phenomenon can be periodic even though it is not connected to motion in a circle. For example, here is a graph of the water level in Bridgeport, Connecticut, over a 50 hour period in 2018.



Notice that each day (or each 24 hour period) there are two tides, a small one where the water goes up to a little less than 3 feet and then a bigger one when the tide goes up to a little more than 3 feet. Since there are two tides per day, the period for this graph is about 12 hours. The data begins about 1 hour before the tide is at the 0 value. Since $\sin(0) = 0$, this would make the sine a good choice for modeling the tide.

Putting together all of our information gives model $f(h) = 3\sin\left(\frac{2\pi}{12}(h-1)\right)$, where h measures hours since midnight on September 1.



Notice that:

- The coefficient of 3 is the amplitude, which averages out the bigger and smaller tides.
- $\frac{2\pi}{12}$ makes the period 12 hours.
- -1 translates the sine graph to the right by 1 hour to so it has a value of 0 at about 1 hour after midnight.