Lesson 14: Completing the Square (Part 3)

- Let's complete the square for some more complicated expressions.

14.1: Perfect Squares in Two Forms

Elena says, "$(x+3)^2$ can be expanded into x^2+6x+9. Likewise, $(2x+3)^2$ can be expanded into $4x^2+6x+9$."

Find an error in Elena's statement and correct the error. Show your reasoning.

14.2: Perfect in A Different Way

1. Write each expression in standard form:

    a. $(4x+1)^2$

    b. $(5x-2)^2$

    c. $(\frac{1}{2}x+7)^2$

    d. $(3x+n)^2$

    e. $(kx+m)^2$

2. Decide if each expression is a perfect square. If so, write an equivalent expression of the form $(kx+m)^2$. If not, suggest one change to turn it into a perfect square.

    a. $4x^2+12x+9$

    b. $4x^2+8x+25$

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14.3: When All the Stars Align

1. Find the value of c to make each expression in the left column a perfect square in standard form. Then, write an equivalent expression in the form of squared factors. In the last row, write your own pair of equivalent expressions.

standard form $(ax^2 + bx + c)$	squared factors $(kx + m)^2$
$100x^2 + 80x + c$	
$36x^2 - 60x + c$	
$25x^2 + 40x + c$	
$0.25x^2 - 14x + c$	

2. Solve each equation by completing the square:

$25x^2 + 40x = -12$

$36x^2 - 60x + 10 = -6$

14.4: Putting Stars into Alignment

Here are three methods for solving $3x^2 + 8x + 5 = 0$.

Try to make sense of each method.

Method 1:

$$3x^2 + 8x + 5 = 0$$
$$(3x + 5)(x + 1) = 0$$

$$x = -\frac{5}{3} \quad \text{or} \quad x = -1$$

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Method 2:

$$\begin{aligned} 3x^2 + 8x + 5 &= 0 \\ 9x^2 + 24x + 15 &= 0 \\ (3x)^2 + 8(3x) + 15 &= 0 \\ U^2 + 8U + 15 &= 0 \\ (U+5)(U+3) &= 0 \end{aligned}$$

$$\begin{aligned} U &= \text{-}5 \quad \text{or} \quad U = \text{-}3 \\ 3x &= \text{-}5 \quad \text{or} \quad 3x = \text{-}3 \\ x &= \text{-}\frac{5}{3} \quad \text{or} \quad x = \text{-}1 \end{aligned}$$

Method 3:

$$\begin{aligned} 3x^2 + 8x + 5 &= 0 \\ 9x^2 + 24x + 15 &= 0 \\ 9x^2 + 24x + 16 &= 1 \\ (3x+4)^2 &= 1 \end{aligned}$$

$$\begin{aligned} 3x + 4 = 1 \quad &\text{or} \quad 3x + 4 = \text{-}1 \\ x = \text{-}1 \quad &\text{or} \quad x = \text{-}\frac{5}{3} \end{aligned}$$

Once you understand the methods, use each method at least one time to solve these equations.

1. $5x^2 + 17x + 6 = 0$

2. $6x^2 + 19x = \text{-}10$

3. $8x^2 - 33x + 4 = 0$

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4. $8x^2 - 26x =$ -21

5. $10x^2 + 37x = 36$

6. $12x^2 + 20x - 77 = 0$

Are you ready for more?

Find the solutions to $3x^2 - 6x + \frac{9}{4} = 0$. Explain your reasoning.

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Lesson 14 Summary

In earlier lessons, we worked with perfect squares such as $(x+1)^2$ and $(x-5)(x-5)$. We learned that their equivalent expressions in standard form follow a predictable pattern:

- In general, $(x+m)^2$ can be written as $x^2+2mx+m^2$.
- If a quadratic expression of the form ax^2+bx+c is a perfect square, and the value of a is 1, then the value of b is $2m$, and the value of c is m^2 for some value of m.

In this lesson, the variable in the factors being squared had a coefficient other than 1, for example $(3x+1)^2$ and $(2x-5)(2x-5)$. Their equivalent expression in standard form also followed the same pattern we saw earlier.

squared factors	standard form
$(3x+1)^2$	$(3x)^2+2(3x)(1)+1^2$ or $9x^2+6x+1$
$(2x-5)^2$	$(2x)^2+2(2x)(\text{-}5)+(\text{-}5)^2$ or $4x^2-20x+25$

In general, $(kx+m)^2$ can be written as:

$(kx)^2+2(kx)(m)+m^2$ or $k^2x^2+2kmx+m^2$

If a quadratic expression is of the form ax^2+bx+c, then:

- the value of a is k^2
- the value of b is $2km$
- the value of c is m^2

We can use this pattern to help us complete the square and solve equations when the squared term x^2 has a coefficient other than 1—for example: $16x^2+40x=11$.

What constant term c can we add to make the expression on the left of the equal sign a perfect square? And how do we write this expression as squared factors?

- 16 is 4^2, so the squared factors could be $(4x+m)^2$.
- 40 is equal to $2(4m)$, so $2(4m)=40$ or $8m=40$. This means that $m=5$.
- If c is m^2, then $c=5^2$ or $c=25$.

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- So the expression $16x^2 + 40x + 25$ is a perfect square and is equivalent to $(4x + 5)^2$.

Let’s solve the equation $16x^2 + 40x = 11$ by completing the square!

$$\begin{aligned} 16x^2 + 40x &= 11 \\ 16x^2 + 40x + 25 &= 11 + 25 \\ (4x + 5)^2 &= 36 \end{aligned}$$

$$\begin{array}{rcl} 4x + 5 = 6 & \text{or} & 4x + 5 = \text{-}6 \\ 4x = 1 & \text{or} & 4x = \text{-}11 \\ x = \frac{1}{4} & \text{or} & x = \text{-}\frac{11}{4} \end{array}$$

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