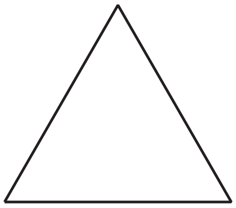
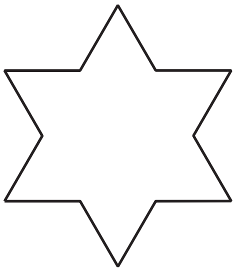
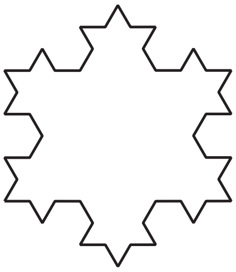
Unit 2 Lesson 25: Summing Up

1 Notice & Wonder: A Snowflake’s Return (Warm up)

Student Task Statement

What do you notice? What do you wonder?

iteration	total number of triangles added since the first
0 	0
1 	3
2 	$3 + 3(4) = 15$
3	$3 + 3(4) + 3(4)^2 = 63$
n	$3 + 3(4) + 3(4)^2 + \ldots + 3(4)^{n-1}$

2 A Geometric Addition Formula

Student Task Statement

Earlier, we learned that the nth term of a geometric sequence with an initial value of a and a common ratio of r is $a(r^{n-1})$.

For a Koch Snowflake, it turns out that we can find the number of triangles added on at each iteration by having $a = 3$ and $r = 4$. The sum s of the first n terms in this geometric sequence tell us how many triangles total make up the nth iteration of the snowflake

$$s = 3 + 3(4) + 3(4^2) + \ldots + 3(4^{n-1})$$

More generally, the sum of the first n terms of any geometric sequence can be expressed as

$$s = a + a(r) + a(r^2) + \ldots + a(r^{n-1})$$

or

$$s = a(1 + r + r^2 + \ldots + r^{n-1})$$

1. What would happen if we multiplied each side of this equation by $(1 - r)$? (hint: $(x - 1)(x^3 + x^2 + x + 1) = x^4 - 1$.)
2. Rewrite the new equation in the form of $s =$.
3. Use this new formula to calculate how many triangles after the original are in the first 5, 10, and 15 iterations of the Koch Snowflake.

3 The Sum of Antibiotics

Student Task Statement

Han is prescribed a course of antibiotics for an infection. He is told to take a 150 mg dose of the antibiotic regularly every 12 hours for 15 days. Han is curious about the antibiotic and learns that at the end of the 12 hours, only 5% of the dose is still in his body.

1. How much of the antibiotic is in the body right after the first, second, and third doses?
2. When will the total amount of the antibiotic in Han be highest over the course of the 15 day treatment? Explain your reasoning.