Lesson 16: The Quadratic Formula

- Let's learn a formula for finding solutions to quadratic equations.

16.1: Evaluate It

Each expression represents two numbers. Evaluate the expressions and find the two numbers.

1. $1 \pm \sqrt{49}$
2. $\frac{8 \pm 2}{5}$
3. $\pm\sqrt{(-5)^2 - 4 \cdot 4 \cdot 1}$
4. $\frac{-18 \pm \sqrt{36}}{2 \cdot 3}$

16.2: Pesky Equations

Choose one equation to solve, either by rewriting it in factored form or by completing the square. Be prepared to explain your choice of method.

1. $x^2 - 2x - 1.25 = 0$
2. $5x^2 + 9x - 44 = 0$
3. $x^2 + 1.25x = 0.375$
4. $4x^2 - 28x + 29 = 0$

16.3: Meet the Quadratic Formula

Here is a formula called the **quadratic formula**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The formula can be used to find the solutions to any quadratic equation in the form of $ax^2 + bx + c = 0$, where a, b, and c are numbers and a is not 0.

This example shows how it is used to solve $x^2 - 8x + 15 = 0$, in which $a = 1$, $b = -8$, and $c = 15$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{original equation}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)} \qquad \text{substitute the values of } a, b, \text{ and } c$$

$$x = \frac{8 \pm \sqrt{64 - 60}}{2} \qquad \text{evaluate each part of the expression}$$

$$x = \frac{8 \pm \sqrt{4}}{2}$$

$$x = \frac{8 \pm 2}{2}$$

$$x = \frac{10}{2} \quad \text{or} \quad x = \frac{6}{2}$$

$$x = 5 \quad \text{or} \quad x = 3$$

Here are some quadratic equations and their solutions. Use the quadratic formula to show that the solutions are correct.

1. $x^2 + 4x - 5 = 0$. The solutions are $x = -5$ and $x = 1$.

2. $x^2 + 7x + 12 = 0$. The solutions are $x = -3$ and $x = -4$.

3. $x^2 + 10x + 18 = 0$. The solutions are $x = -5 \pm \frac{\sqrt{28}}{2}$.

4. $x^2 - 8x + 11 = 0$. The solutions are $x = 4 \pm \frac{\sqrt{20}}{2}$.

5. $9x^2 - 6x + 1 = 0$. The solution is $x = \frac{1}{3}$.

6. $6x^2 + 9x - 15 = 0$. The solutions are $x = -\frac{5}{2}$ and $x = 1$.

Are you ready for more?

1. Use the quadratic formula to solve $ax^2 + c = 0$. Let's call the resulting equation P.

2. Solve the equation $3x^2 - 27 = 0$ in two ways, showing your reasoning for each:
    - Without using any formulas.
    - Using equation P.

3. Check that you got the same solutions using each method.

4. Use the quadratic formula to solve $ax^2 + bx = 0$. Let's call the resulting equation Q.

5. Solve the equation $2x^2 + 5x = 0$ in two ways, showing your reasoning for each:
    - Without using any formulas.
    - Using equation Q.

6. Check that you got the same solutions using each method.

Lesson 16 Summary

We have learned a couple of methods for solving quadratic equations algebraically:

- by rewriting the equation as $\text{factored form} = 0$ and using the zero product property
- by completing the square

Some equations can be solved quickly with one of these methods, but many cannot. Here is an example: $5x^2 - 3x - 1 = 0$. The expression on the left cannot be rewritten in factored form with rational coefficients. Because the coefficient of the squared term is not a perfect square, and the coefficient of the linear term is an odd number, completing the square would be inconvenient and would result in a perfect square with fractions.

The **quadratic formula** can be used to find the solutions to any quadratic equation, including those that are tricky to solve with other methods.

For an equation of the form $ax^2 + bx + c = 0$, where a, b, and c are numbers and $a \neq 0$, the solutions are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $5x^2 - 3x - 1 = 0$, we see that $a = 5$, $b = -3$, and $c = -1$. Let's solve it!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{the quadratic formula}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)} \qquad \text{substitute the values of } a, b, \text{and } c$$

$$x = \frac{3 \pm \sqrt{9 + 20}}{10} \qquad \text{evaluate each part of the expression}$$

$$x = \frac{3 \pm \sqrt{29}}{10}$$

A calculator gives approximate solutions of 0.84 and -0.24 for $\frac{3+\sqrt{29}}{10}$ and $\frac{3-\sqrt{29}}{10}$.

We can also use the formula for simpler equations like $x^2 - 9x + 8 = 0$, but it may not be the most efficient way. If the quadratic expression can be easily rewritten in factored form or made into a perfect square, those methods may be preferable. For example, rewriting $x^2 - 9x + 8 = 0$ as $(x-1)(x-8) = 0$ immediately tells us that the solutions are 1 and 8.