Unit 2 Lesson 24: Polynomial Identities (Part 2)

1 Revisiting an Old Theorem (Warm up)

Student Task Statement

Instructions to make a right triangle:

- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

Follow these instructions to make a few different triangles. Do you think the instructions always produce a right triangle? Be prepared to explain your reasoning.

2 Theorems and Identities

Student Task Statement

Here are the instructions to make a right triangle from earlier:

- Choose two integers.
- Make one side length equal to the sum of the squares of the two integers.
- Make one side length equal to the difference of the squares of the two integers.
- Make one side length equal to twice the product of the two integers.

1. Using a and b for the two integers, write expressions for the three side lengths.
2. Why do these instructions make a right triangle?

3 Identifying Identities (Optional)

Student Task Statement

Here is a list of equations. Circle all the equations that are identities. Be prepared to explain your reasoning.

1. $a = \text{-}a$
2. $a^2 + 2ab + b^2 = (a + b)^2$
3. $a^2 - 2ab + b^2 = (a - b)^2$
4. $a^2 - b^2 = (a - b)(a - b)$
5. $(a + b)(a^2 - ab + b^2) = a^3 - b^3$
6. $(a - b)^3 = a^3 - b^3 - 3ab(a + b)$
7. $a^2(a - b)^4 - b^2(a - b)^4 = (a - b)^5(a + b)$

4 Egyptian Fractions

Student Task Statement

In Ancient Egypt, all non-unit fractions were represented as a sum of distinct unit fractions. For example, $\frac{4}{9}$ would have been written as $\frac{1}{3} + \frac{1}{9}$ (and not as $\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$ or any other form with the same unit fraction used more than once). Let's look at some different ways we can rewrite $\frac{2}{15}$ as the sum of distinct unit fractions.

1. Use the formula $\frac{2}{d} = \frac{1}{d} + \frac{1}{2d} + \frac{1}{3d} + \frac{1}{6d}$ to rewrite the fraction $\frac{2}{15}$, then show that this formula is an identity.
2. Another way to rewrite fractions of the form $\frac{2}{d}$ is given by the identity $\frac{2}{d} = \frac{1}{d} + \frac{1}{d+1} + \frac{1}{d(d+1)}$. Use it to re-write the fraction $\frac{2}{15}$, then show that it is an identity.