## Lesson 14: More Arithmetic with Complex Numbers

* Let’s practice adding, subtracting, and multiplying complex numbers.

### 14.1: Which One Doesn’t Belong: Complex Expressions

Which one doesn’t belong?

A. $i^{2}$

B. $\left(1+i\right)+\left(1−i\right)$

C. $\left(1+i\right)^{2}$

D. $\left(1+i\right)\left(1−i\right)$

### 14.2: Powers of $i$

1. Write each power of $i$ in the form $a+bi$, where $a$ and $b$ are real numbers. If $a$ or $b$ is zero, you can ignore that part of the number. For example, $0+3i$ can simply be expressed as $3i$.
* $i^{0}$
* $i^{1}$
* $i^{2}$
* $i^{3}$
* $i^{4}$
* $i^{5}$
* $i^{6}$
* $i^{7}$
* $i^{8}$
1. What is $i^{100}$? Explain your reasoning.
2. What is $i^{38}$? Explain your reasoning.

#### Are you ready for more?

1. Write each power of $1+i$ in the form $a+bi$, where $a$ and $b$ are real numbers. If $a$ or $b$ is zero, you can ignore that part of the number. For example, $0+3i$ can simply be expressed as $3i$.
	1. $\left(1+i\right)^{0}$
	2. $\left(1+i\right)^{1}$
	3. $\left(1+i\right)^{2}$
	4. $\left(1+i\right)^{3}$
	5. $\left(1+i\right)^{4}$
	6. $\left(1+i\right)^{5}$
	7. $\left(1+i\right)^{6}$
	8. $\left(1+i\right)^{7}$
	9. $\left(1+i\right)^{8}$
2. Compare and contrast the powers of $1+i$ with the powers of $i$. What is the same? What is different?

### 14.3: Add 'Em Up (or Subtract or Multiply)

For each row, your partner and you will each rewrite an expression so it has the form $a+bi$, where $a$ and $b$ are real numbers. You and your partner should get the same answer. If you disagree, work to reach agreement.

| partner A | partner B |
| --- | --- |
| $\left(7+9i\right)+\left(3−4i\right)$ | $5i\left(1−2i\right)$ |
| $2i\left(3+4i\right)$ | $\left(1+2i\right)−\left(9−4i\right)$ |
| $\left(4−3i\right)\left(4+3i\right)$ | $\left(5+i\right)+\left(20−i\right)$ |
| $\left(2i\right)^{4}$ | $\left(3+i\sqrt{7}\right)\left(3−i\sqrt{7}\right)$ |
| $\left(1+i\sqrt{5}\right)−\left(-7−i\sqrt{5}\right)$ | $\left(-2i\right)\left(-\sqrt{5}+4i\right)$ |
| $\left(\frac{1}{2}i\right)\left(\frac{1}{3}i\right)\left(\frac{3}{4}i\right)$ | $\left(\frac{1}{2}i\right)^{3}$ |

### Lesson 14 Summary

Suppose we want to write the product $\left(3+5i\right)\left(7−2i\right)$ in the form $a+bi$, where $a$ and $b$ are real numbers. For example, we might want to compare our solution with a partner’s, and having answers in the same form makes that easier. Using the distributive property,

$\begin{matrix}\left(3+5i\right)\left(7−2i\right)&=21−6i+35i−10i^{2}\\&=21+29i−10\left(-1\right)\\&=21+29i+10\\&=31+29i\end{matrix}$

Keeping track of the negative signs is especially important since it is easy to mix up the fact that $i^{2}=-1$ with the fact that $-i^{2}=-\left(-1\right)=1$.

Next, suppose we want to write the difference $\left(-6+3i\right)−\left(2−4i\right)$ as a single complex number in the form $a+bi$. Distributing the negative and combining like terms, we get:

$\begin{matrix}\left(-6+3i\right)−\left(2−4i\right)&=-6+3i−2−\left(-4i\right)\\&=-8+3i+4i\\&=-8+7i\end{matrix}$

Again, it is important to be precise with negative signs. It is a common mistake to just subtract $4i$ rather than subtracting $-4i$.



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