Lesson 9: Dealing with Negative Numbers

Let's show that doing the same to each side works for negative numbers too.

9.1: Which One Doesn't Belong: Rational Number Arithmetic

Which equation doesn't belong?

$15 = -5 \cdot -3$

$4 - -2 = 6$

$2 + -5 = -3$

$-3 \cdot -4 = -12$

9.2: Old and New Ways to Solve

Solve each equation. Be prepared to explain your reasoning.

1. $x + 6 = 4$
2. $x - -4 = -6$
3. $2(x - 1) = -200$
4. $2x + -3 = -23$

9.3: Keeping It True

Here are some equations that all have the same solution.

$$x = \text{-}6$$
$$x - 3 = \text{-}9$$
$$\text{-}9 = x - 3$$
$$900 = \text{-}100(x - 3)$$
$$900 = (x - 3) \cdot (\text{-}100)$$
$$900 = \text{-}100x + 300$$

1. Explain how you know that each equation has the same solution as the previous equation. Pause for discussion before moving to the next question.

2. Keep your work secret from your partner. Start with the equation $\text{-}5 = x$. Do the same thing to each side at least three times to create an equation that has the same solution as the starting equation. Write the equation you ended up with on a slip of paper, and trade equations with your partner.

3. See if you can figure out what steps they used to transform $\text{-}5 = x$ into their equation. When you think you know, check with them to see if you are right.

Lesson 9 Summary

When we want to find a solution to an equation, sometimes we just think about what value in place of the variable would make the equation true. Sometimes we perform the same operation on each side (for example, subtract the same amount from each side). The balanced hangers helped us to understand that doing the same thing to each side of an equation keeps the equation true.

Since negative numbers are just numbers, then doing the same thing to each side of an equation works for negative numbers as well. Here are some examples of equations that have negative numbers and steps you could take to solve them.

Example:

$$\begin{aligned} 2(x-5) &= \text{-}6 \\ \tfrac{1}{2} \cdot 2(x-5) &= \tfrac{1}{2} \cdot (\text{-}6) && \text{multiply each side by } \tfrac{1}{2} \\ x-5 &= \text{-}3 \\ x-5+5 &= \text{-}3+5 && \text{add 5 to each side} \\ x &= 2 \end{aligned}$$

Example:

$$\begin{aligned} \text{-}2x + \text{-}5 &= 6 \\ \text{-}2x + \text{-}5 - \text{-}5 &= 6 - \text{-}5 && \text{subtract -5 from each side} \\ \text{-}2x &= 11 \\ \text{-}2x \div \text{-}2 &= 11 \div \text{-}2 && \text{divide each side by -2} \\ x &= \text{-}\tfrac{11}{2} \end{aligned}$$

Doing the same thing to each side maintains equality even if it is not helpful to solving for the unknown amount. For example, we could take the equation $\text{-}3x + 7 = \text{-}8$ and add -2 to each side:

$$\begin{aligned} \text{-}3x + 7 &= \text{-}8 \\ \text{-}3x + 7 + \text{-}2 &= \text{-}8 + \text{-}2 && \text{add -2 to each side} \\ \text{-}3x + 5 &= \text{-}10 \end{aligned}$$

If $\text{-}3x + 7 = \text{-}8$ is true then $\text{-}3x + 5 = \text{-}10$ is also true, but we are no closer to a solution than we were before adding -2. We can use moves that maintain equality to make new equations that all have the same solution. Helpful combinations of moves will eventually lead to an equation like $x = 5$, which gives the solution to the original equation (and every equation we wrote in the process of solving).