Family Support Materials

Fractions and Decimals

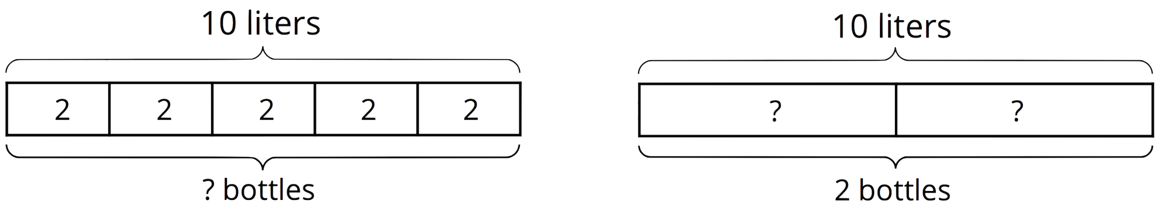
Making Sense of Division

Family Support Materials 1

This week, your student will be thinking about the meanings of division to prepare to learn about division of fraction. Suppose we have 10 liters of water to divide into equal-size groups. We can think of the division $10 \div 2$ in two ways, or as the answer to two questions:

- “How many bottles can we fill with 10 liters if each bottle has 2 liters?”
- “How many liters are in each bottle if we divide 10 liters into 2 bottles?”

Here are two diagrams to show the two interpretations of $10 \div 2$:



In both cases, the answer to the question is 5, but it could either mean “there are 5 bottles with 2 liters in each” or “there are 5 liters in each of the 2 bottles.”

Here is a task to try with your student:

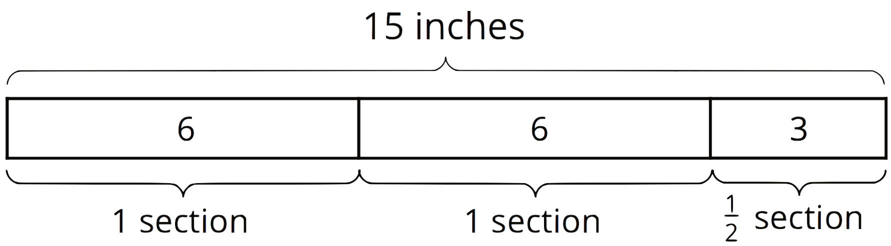
1. Write two different questions we can ask about $15 \div 6$.
2. Estimate the answer: Is it less than 1, equal to 1, or greater than 1? Explain your estimate.
3. Find the answer to one of the questions you wrote. It might help to draw a picture.

Solution:

1. Questions vary. Sample questions:
    - A ribbon that is 15 inches long is divided into 6 equal sections. How long (in inches) is each section?
    - A ribbon that is 15 inches is divided into 6-inch sections. How many sections are there?

2. Greater than 1. Sample explanations:
   - $12 \div 6$ is 2, so $15 \div 6$ must be greater than 2.
   - If we divide 15 into 15 groups ($15 \div 15$), we get 1. So if we divide 15 into 6, which is a smaller number of groups, the amount in each group must be greater than 1.

3. $2\frac{1}{2}$. Sample diagram:

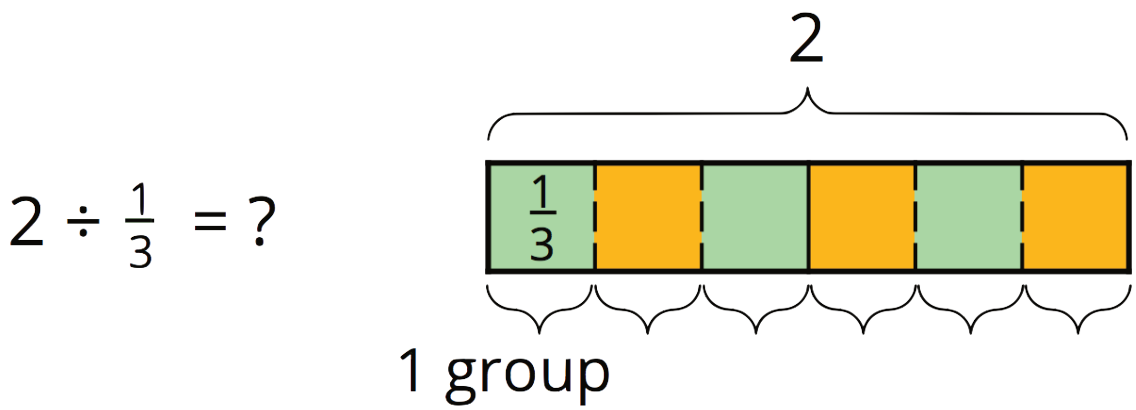


Dividing Fractions

Family Support Materials 2

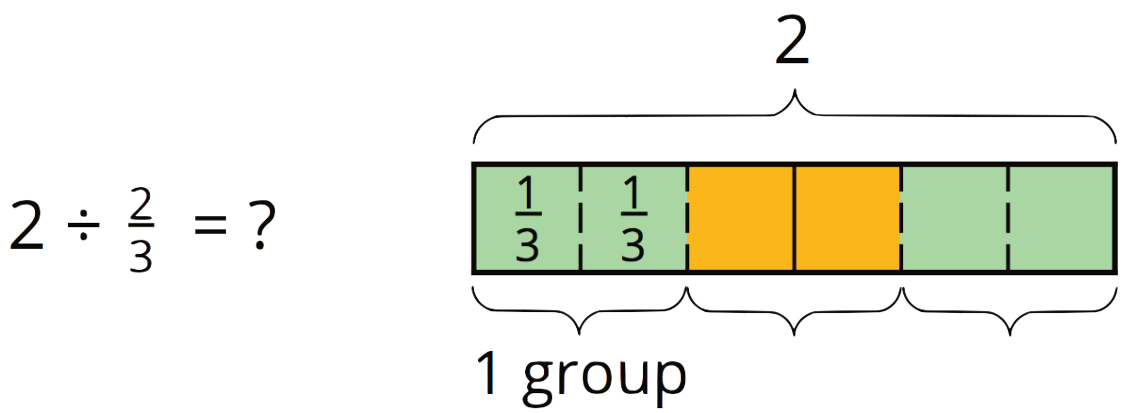
Many people have learned that to divide a fraction, we "invert and multiply." This week, your student will learn why this works by studying a series of division statements and diagrams such as these:

- $2 \div \frac{1}{3} = ?$ can be viewed as "how many $\frac{1}{3}$s are in 2?"



  Because there are 3 thirds in 1, there are $(2 \cdot 3)$ or 6 thirds in 2. So dividing 2 by $\frac{1}{3}$ has the same outcome as multiplying 2 by 3.

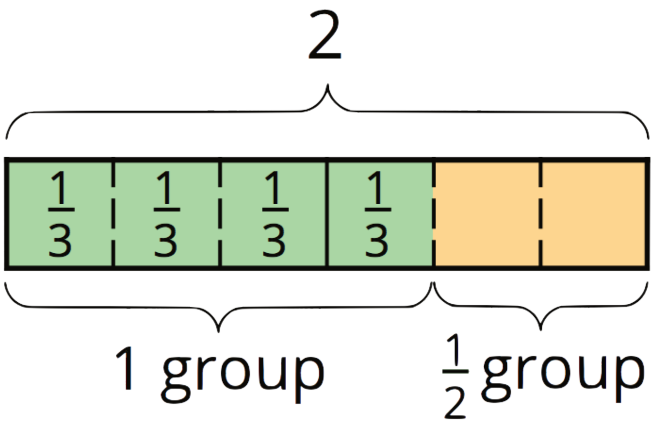
- $2 \div \frac{2}{3} = ?$ can be viewed as "how many $\frac{2}{3}$s are in 2?"



  We already know that there are $(2 \cdot 3)$ or 6 thirds in 2. To find how many $\frac{2}{3}$s are in 2, we need to combine every 2 of the thirds into a group. Doing this results in half as many groups. So $2 \div \frac{2}{3} = (2 \cdot 3) \div 2$, which equals 3.

- $2 \div \frac{4}{3} = ?$ can be viewed as "how many $\frac{4}{3}$s are in 2?"

$2 \div \frac{4}{3} = ?$



Again, we know that there are $(2 \cdot 3)$ thirds in 2. To find how many $\frac{4}{3}$s are in 2, we need to combine every 4 of the thirds into a group. Doing this results in one fourth as many groups. So $2 \div \frac{4}{3} = (2 \cdot 3) \div 4$, which equals $1\frac{1}{2}$.

Notice that each division problem above can be answered by multiplying 2 by the denominator of the divisor and then dividing it by the numerator. So $2 \div \frac{a}{b}$ can be solved with $2 \cdot b \div a$, which can also be written as $2 \cdot \frac{b}{a}$. In other words, dividing 2 by $\frac{a}{b}$ has the same outcome as multiplying 2 by $\frac{b}{a}$. The fraction in the divisor is “inverted” and then multiplied.

Here is a task to try with your student:

1. Find each quotient. Show your reasoning.
   a. $3 \div \frac{1}{7}$

   b. $3 \div \frac{3}{7}$

   c. $3 \div \frac{6}{7}$

   d. $\frac{3}{7} \div \frac{6}{7}$

2. A sack of flour weighs 4 pounds. A grocer is distributing the flour into equal-size bags.
   a. Write a question that $4 \div \frac{2}{5} = ?$ could represent in this situation.

   b. Find the answer. Explain or show your reasoning.

Solution:

1. a. 21. Sample reasoning: $3 \div \frac{1}{7} = 3 \cdot \frac{7}{1} = 21$

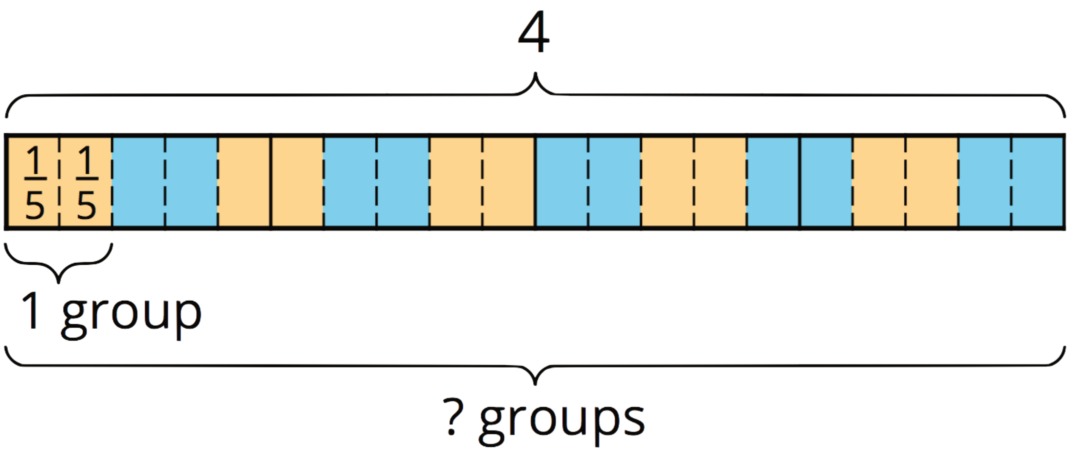
b. 7. Sample reasoning: $3 \div \frac{3}{7} = 3 \cdot \frac{7}{3} = 7$

c. $3\frac{1}{2}$. Sample reasoning: $3 \div \frac{1}{7} = 3 \cdot \frac{7}{6} = \frac{7}{2}$. The fraction $\frac{6}{7}$ is two times $\frac{3}{7}$, so there are half as many $\frac{6}{7}$s in 3 as there are $\frac{3}{7}$s.

d. $\frac{1}{2}$. Sample reasoning: $\frac{3}{7} \div \frac{6}{7} = \frac{3}{7} \cdot \frac{7}{6} = \frac{3}{6}$

2. a. 4 pounds of flour are divided equally into bags of $\frac{2}{5}$-pound each. How many bags will there be?

   b. 10 bags. Sample reasoning: Break every 1 pound into fifths and then count how many groups of $\frac{2}{5}$ there are.



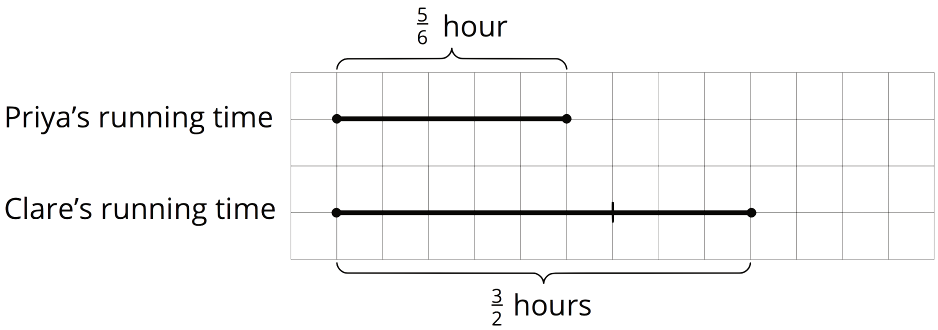
Fractions in Lengths, Areas, and Volumes

Family Support Materials 3

Over the next few days, your student will be solving problems that require multiplying and dividing fractions. Some of these problems will be about comparison. For example:

- If Priya ran for $\frac{5}{6}$ hour and Clare ran for $\frac{3}{2}$ hours, what fraction of Clare's running time was Priya's running time?

  We can draw a diagram and write a multiplication equation to make sense of the situation.

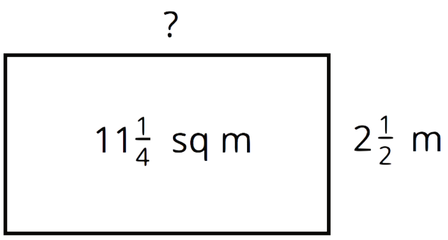


$$\text{(fraction)} \cdot \text{(Clare's time)} = \text{(Priya's time)}$$
$$? \cdot \frac{3}{2} = \frac{5}{6}$$

  We can find the unknown by dividing. $\frac{5}{6} \div \frac{3}{2} = \frac{5}{6} \cdot \frac{2}{3}$, which equals $\frac{10}{18}$. So Priya's running time was $\frac{10}{18}$ or $\frac{5}{9}$ of Clare's.

Other problems your students will solve are related to geometry—lengths, areas, and volumes. For examples:

- What is the length of a rectangular room if its width is $2\frac{1}{2}$ meters and its area is $11\frac{1}{4}$ square meters?



We know that the area of a rectangle can be found by multiplying its length and width ($? \cdot 2\frac{1}{2} = 11\frac{1}{4}$), so dividing $11\frac{1}{4} \div 2\frac{1}{2}$ (or $\frac{45}{4} \div \frac{5}{2}$) will give us the length of the room. $\frac{45}{4} \div \frac{5}{2} = \frac{45}{4} \cdot \frac{2}{5} = \frac{9}{2}$. The room is $4\frac{1}{2}$ meters long.

- What is the volume of a box (a rectangular prism) that is $3\frac{1}{2}$ feet by 10 feet by $\frac{1}{4}$ foot?

  We can find the volume by multiplying the edge lengths. $3\frac{1}{2} \cdot 10 \cdot \frac{1}{4} = \frac{7}{2} \cdot 10 \cdot \frac{1}{4}$, which equals $\frac{70}{8}$. So the volume is $\frac{70}{8}$ or $8\frac{6}{8}$ cubic feet.

Here is a task to try with your student:

1. In the first example about Priya and Clare's running times, how many times as long as Priya's running time was Clare's running time? Show your reasoning.
2. The area of a rectangle is $\frac{20}{3}$ square feet. What is its width if its length is $\frac{4}{3}$ feet? Show your reasoning.

Solution:

1. $\frac{9}{5}$. Sample reasoning: We can write $? \cdot \frac{5}{6} = \frac{3}{2}$ to represent the question "how many times of Priya's running time was Clare's running time?" and then solve by dividing. $\frac{3}{2} \div \frac{5}{6} = \frac{3}{2} \cdot \frac{6}{5} = \frac{18}{10}$. Clare's running time was $\frac{18}{10}$ or $\frac{9}{5}$ as long as Priya's.
2. 5 feet. Sample reasoning: $\frac{20}{3} \div \frac{4}{3} = \frac{20}{3} \cdot \frac{3}{4} = \frac{20}{4} = 5$

Warming Up to Decimals

Family Support Materials 4

This week, your student will add and subtract numbers using what they know about the meaning of the digits. In earlier grades, your student learned that the 2 in 207.5 represents 2 *hundreds*, the 7 represents 7 *ones*, and the 5 represents 5 *tenths*. We add and subtract the digits that correspond to the same units like *hundreds* or *tenths*. For example, to find $10.5 + 84.3$, we add the tens, the ones, and the tenths separately, so $10.5 + 84.3 = 90 + 4 + 0.8 = 94.8$.

Any time we add digits and the sum is greater than 10, we can "bundle" 10 of them into the next higher unit. For example, $0.9 + 0.3 = 1.2$.

To add whole numbers and decimal numbers, we can arrange $0.921 + 4.37$ vertically, aligning the decimal points, and find the sum. This is a convenient way to be sure we are adding digits that correspond to the same units. This also makes it easy to keep track when we bundle 10 units into the next higher unit (some people call this "carrying").

$$\begin{array}{r} {\scriptstyle 1} \\ 0.921 \\ +\ 4.37 \\ \hline 5.291 \end{array}$$

Here is a task to try with your student:

Find the value of $6.54 + 0.768$.

Solution: 7.308. Sample explanation: there are 8 thousandths from 0.768. Next, the 4 hundredths from 6.54 and 6 hundredths from 0.768 combined make 1 tenth. Together with the 5 tenths from 6.54 and the 7 tenths from 0.768 this is 13 tenths total or 1 and 3 tenths. In total, there are 7 ones, 3 tenths, no hundredths, and 8 thousandths.

Dividing Decimals

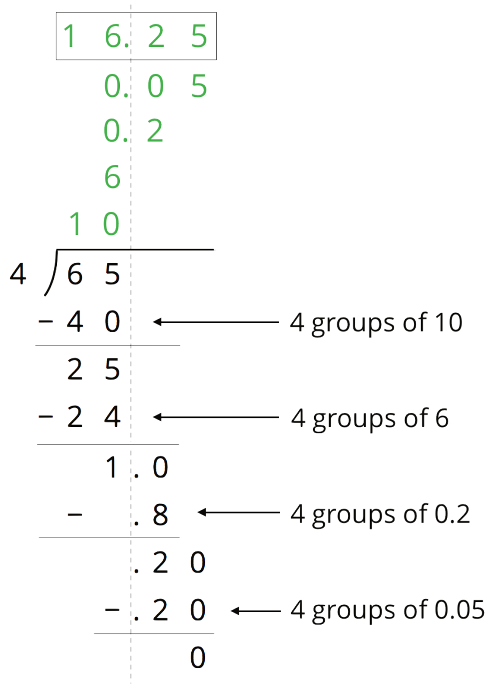
Family Support Materials 5

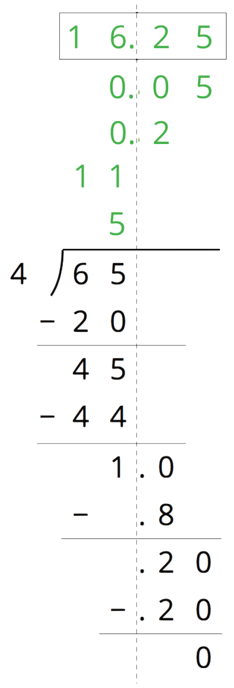
This week, your student will divide whole numbers and decimals. We can think about division as breaking apart a number into equal-size groups.

For example, consider $65 \div 4$. We can image that we are sharing 65 grams of gold equally among 4 people. Here is one way to think about this:

- First give everyone 10 grams. Then 40 grams have been shared out, and 25 grams are left over. We can see this in the first example.
- If we give everyone 6 more grams, then 24 grams have been shared out, and 1 gram is left.
- If we give everyone 0.2 more grams, then 0.8 grams are shared out and 0.2 grams are left.
- If everyone gets 0.05 more grams next, then all of the gold has been shared equally.

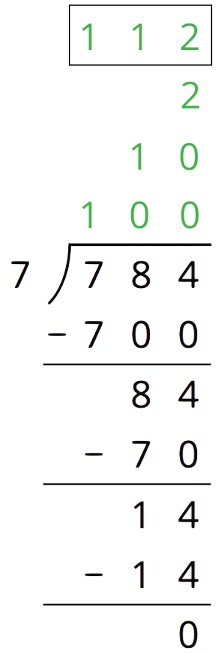
Everyone gets $10 + 6 + 0.2 + 0.05 = 16.25$ grams of gold.





The calculation on the right shows different intermediate steps, but the quotient is the same. This approach is called the **partial quotients** method for dividing.

Here is a task to try with your student:



Here is how Jada found $784 \div 7$ using the partial quotient method.

1. In the calculation, what does the subtraction of 700 represent?

2. Above the dividend 784, we see the numbers 100, 10, and 2. What do they represent?

3. How can we check if 112 if the correct quotient for $784 \div 7$?

Solution

1. Subtraction of 7 groups of 100 from 784.

2. 100, 10, and 2 are the amounts distributed into each group over 3 rounds of dividing.

3. We can multiply $7 \cdot 112$ and see if it produces 784.