Lesson 12: The Number e

- Let's learn about the number e.

12.1: Matching Situations and Equations

Match each equation to a situation it represents. Be prepared to explain how you know. Not all equations have a match.

$f(t) = 400 \cdot (0.5)^{0.1t}$

$g(t) = 400 \cdot (1.25)^{0.1t}$

$h(t) = 400 \cdot (0.75)^{0.1t}$

$j(t) = 400 \cdot (2)^{10t}$

$k(t) = 400 \cdot (2)^{0.1t}$

1. A scientist begins an experiment with 400 bacteria in a petri dish. The population doubles every 10 hours. The function gives the number of bacteria t hours since the experiment began.

2. A patient takes 400 mg of a medicine. The amount of medicine in her bloodstream decreases by 25% every 10 hours. The function gives the amount of medicine left in her bloodstream after t hours of taking the medicine.

3. The half-life of a radioactive element is 10 years. There are 400 g of the element in a sample when it is first studied. The function gives the amount of the element remaining t years later.

4. In a lake, the population of a species of fish is 400. The population is expected to grow by 25% in the next decade. The function gives the number of fish in the lake t years after it was 400.

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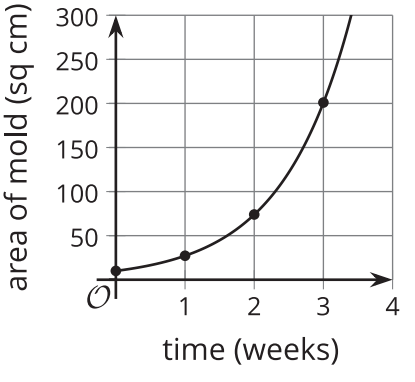
12.2: Notice and Wonder: Moldy Growth

A spot of mold is found on a basement wall. Its area is about 10 square centimeters. Here are three representations of a function that models how the mold is growing.

time (weeks)	area of mold (sq cm)
0	10
1	27
2	74
3	201
4	546

$a(t) = 10 \cdot e^t$



What do you notice? What do you wonder?

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12.3: $(1 + \text{tiny})^{\text{huge}}$

1. Here are some functions. For each function, describe, in words, the outputs for very tiny, positive values of x and for very large values of x.
   $a(x) = 1^x$

   $b(x) = -x$

   $d(x) = \frac{1}{x}$

   $f(x) = \left(\frac{1}{x}\right)^x$

   $g(x) = \left(1 + \frac{1}{x}\right)^x$

   $h(x) = e^x$

   $k(x) = 1 + x$

2. Remember that $e \approx 2.718$. What does the function g have to do with the number e?

3. What do you notice about the relationship between h and k for very small, positive values of x?

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Are you ready for more?

Complete the table to show the value of each expression to the nearest hundred-thousandth. Two entries have already been completed as an example.

x	2^x	e^x	3^x
0.1	1.07177	1.10517	
0.01			
0.001			
0.0001			

What do you notice about the values in the table?

Lesson 12 Summary

Scientists, economists, engineers, and others often use the number e in their mathematical models. What is e?

e is an important constant in mathematics, just like the constant π, which is important in geometry. The value of e is approximately 2.718. Just like π, the number e is irrational, so it can't be represented as a fraction, and its decimal representation never repeats or terminates. The number is named after the 18th-century mathematician Leonhard Euler and is sometimes called *Euler's number*.

e has many useful properties and it arises in situations involving exponential growth or decay, so e often appears in exponential functions. In upcoming lessons, we will work with functions that are expressed using e.

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