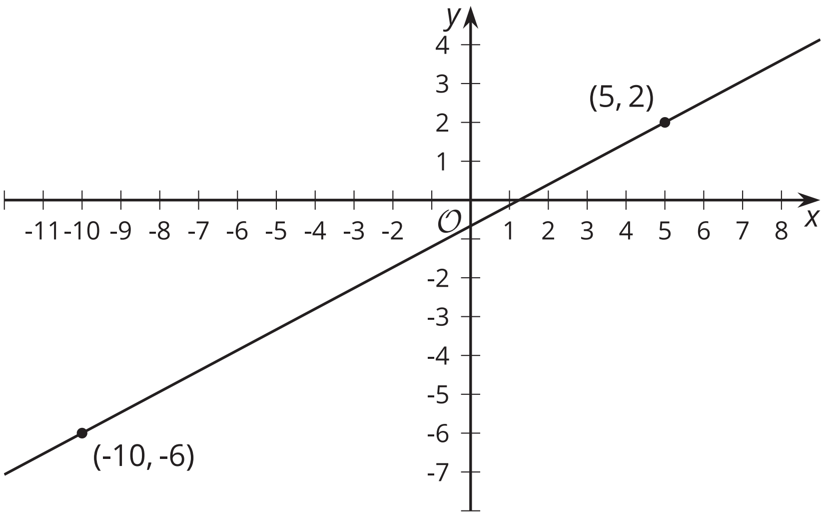
Lesson 9: Equations of Lines

- Let's investigate equations of lines.

9.1: Remembering Slope



The slope of the line in the image is $\frac{8}{15}$. Explain how you know this is true.

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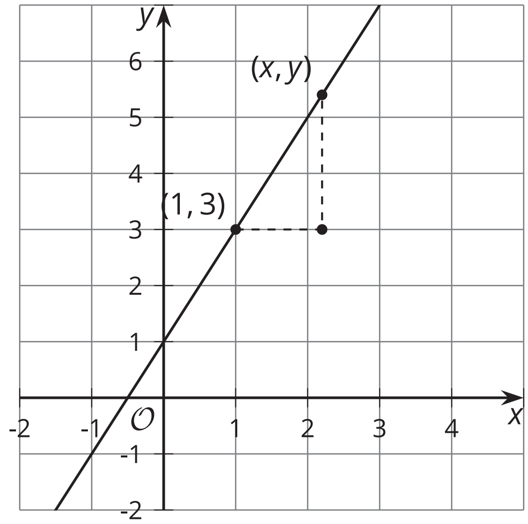
9.2: Building an Equation for a Line

1. The image shows a line.



a. Write an equation that says the slope between the points $(1, 3)$ and (x, y) is 2.

b. Look at this equation: $y - 3 = 2(x - 1)$
How does it relate to the equation you wrote?

2. Here is an equation for another line: $y - 7 = \frac{1}{2}(x - 5)$

a. What point do you know this line passes through?

b. What is the slope of this line?

3. Next, let's write a general equation that we can use for any line. Suppose we know a line passes through a particular point (h, k).

a. Write an equation that says the slope between point (x, y) and (h, k) is m.

b. Look at this equation: $y - k = m(x - h)$. How does it relate to the equation you wrote?

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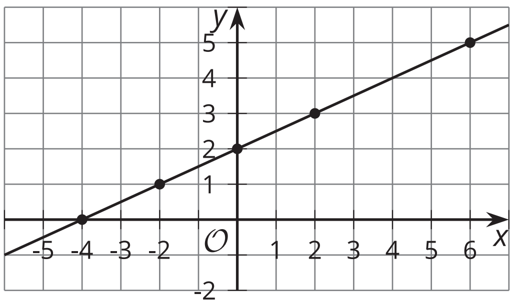
9.3: Using Point-Slope Form

1. Write an equation that describes each line.
   a. the line passing through point $(-2, 8)$ with slope $\frac{4}{5}$

   b. the line passing through point $(0, 7)$ with slope $-\frac{7}{3}$

   c. the line passing through point $(\frac{1}{2}, 0)$ with slope -1

   d. the line in the image



2. Using the structure of the equation, what point do you know each line passes through? What's the line's slope?
   a. $y - 5 = \frac{3}{2}(x + 4)$

   b. $y + 2 = 5x$

   c. $y = -2(x - \frac{5}{8})$

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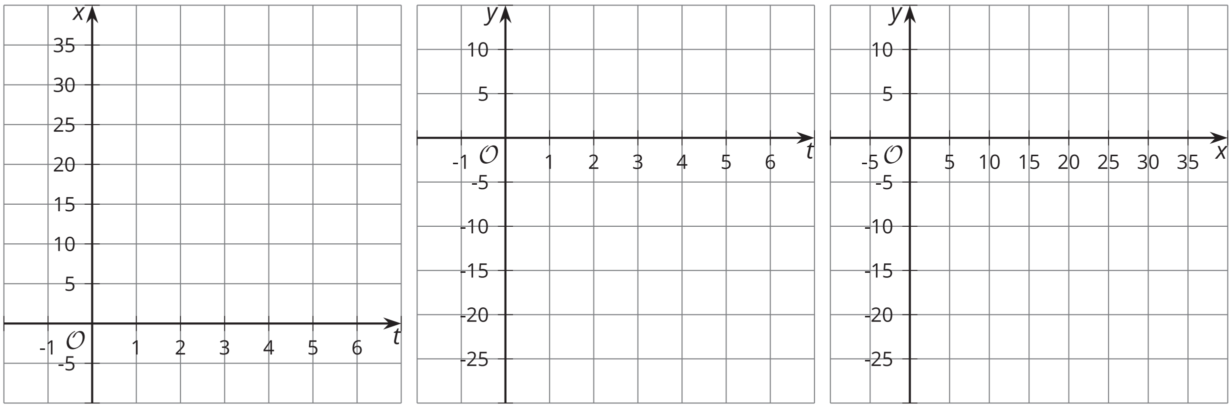
Are you ready for more?

Another way to describe a line, or other graphs, is to think about the coordinates as changing over time. This is especially helpful if we're thinking tracing an object's movement. This example describes the x- and y-coordinates separately, each in terms of time, t.



1. On the first grid, create a graph of $x = 2 + 5t$ for $-2 \leq t \leq 7$ with x on the vertical axis and t on the horizontal axis.
2. On the second grid, create a graph of $y = 3 - 4t$ for $-2 \leq t \leq 7$ with y on the vertical axis and t on the horizontal axis.
3. On the third grid, create a graph of the set of points $(2 + 5t, 3 - 4t)$ for $-2 \leq t \leq 7$ on the xy-plane.

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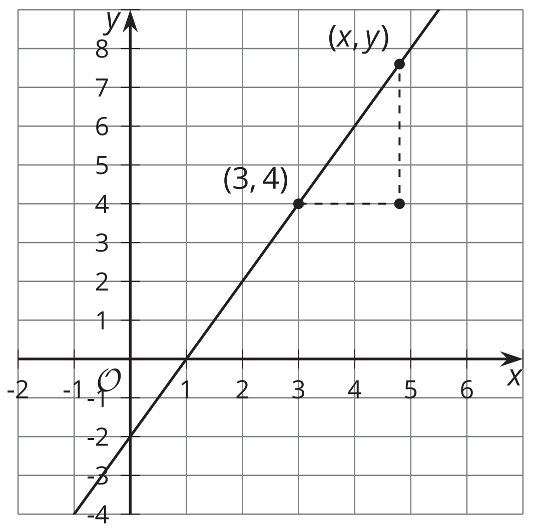
Lesson 9 Summary

The line in the image can be defined as the set of points that have a slope of 2 with the point $(3, 4)$. An equation that says point (x, y) has slope 2 with $(3, 4)$ is $\frac{y-4}{x-3} = 2$. This equation can be rearranged to look like $y - 4 = 2(x - 3)$.



The equation is now in **point-slope form**, or $y - k = m(x - h)$, where:

- (x, y) is any point on the line
- (h, k) is a particular point on the line that we choose to substitute into the equation
- m is the slope of the line

Other ways to write the equation of a line include slope-intercept form, $y = mx + b$, and standard form, $Ax + By = C$.

To write the equation of a line passing through $(3, 1)$ and $(0, 5)$, start by finding the slope of the line. The slope is $-\frac{4}{3}$ because $\frac{5-1}{0-3} = -\frac{4}{3}$. Substitute this value for m to get $y - k = -\frac{4}{3}(x - h)$. Now we can choose any point on the line to substitute for (h, k). If we choose $(3, 1)$, we can write the equation of the line as $y - 1 = -\frac{4}{3}(x - 3)$.

We could also use $(0, 5)$ as the point, giving $y - 5 = -\frac{4}{3}(x - 0)$. We can rearrange the equation to see how point-slope and slope-intercept forms relate, getting $y = -\frac{4}{3}x + 5$. Notice $(0, 5)$ is the y-intercept of the line. The graphs of all 3 of these equations look the same.

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