Lesson 7: Using Graphs to Find Average Rate of Change

- Let's measure how quickly the output of a function changes.

7.1: Temperature Drop

Here are the recorded temperatures at three different times on a winter evening.

time	4 p.m.	6 p.m.	10 p.m.
temperature	$25^\circ F$	$17^\circ F$	$8^\circ F$

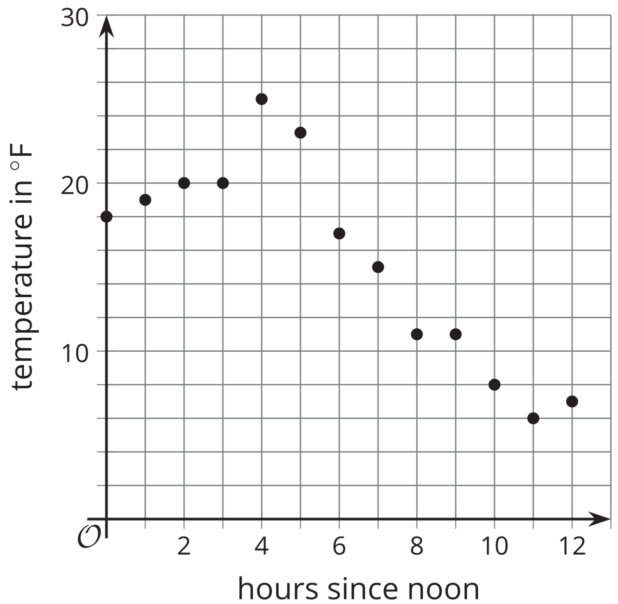
- Tyler says the temperature dropped faster between 4 p.m. and 6 p.m.
- Mai says the temperature dropped faster between 6 p.m. and 10 p.m.

Who do you agree with? Explain your reasoning.

7.2: Drop Some More

The table and graph show a more complete picture of the temperature changes on the same winter day. The function T gives the temperature in degrees Fahrenheit, h hours since noon.

h	$T(h)$
0	18
1	19
2	20
3	20
4	25
5	23
6	17
7	15
8	11
9	11
10	8
11	6
12	7



1. Find the **average rate of change** for the following intervals. Explain or show your reasoning.

    a. between noon and 1 p.m.

    b. between noon and 4 p.m.

    c. between noon and midnight

2. Remember Mai and Tyler's disagreement? Use average rate of change to show which time period—4 p.m. to 6 p.m. or 6 p.m. to 10 p.m.—experienced a faster temperature drop.

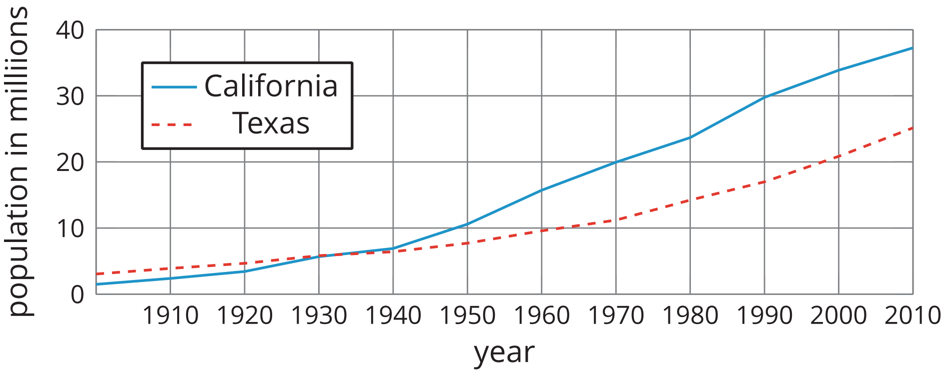
Are you ready for more?

1. Over what interval did the temperature decrease the most rapidly?

2. Over what interval did the temperature increase the most rapidly?

7.3: Populations of Two States

The graphs show the populations of California and Texas over time.



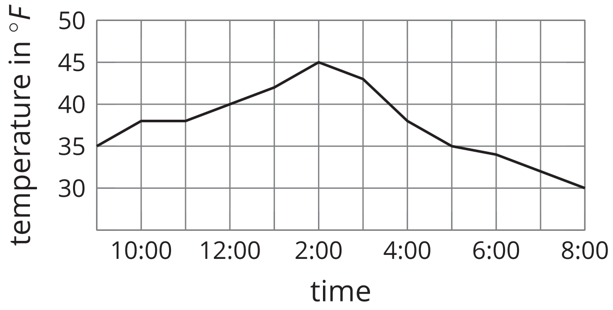
1. a. Estimate the average rate of change in the population in each state between 1970 and 2010. Show your reasoning.

   b. In this situation, what does each rate of change mean?

2. Which state's population grew more quickly between 1900 and 2000? Show your reasoning.

Lesson 7 Summary

Here is a graph of one day's temperature as a function of time.



The temperature was $35^\circ F$ at 9 a.m. and $45^\circ F$ at 2 p.m., an increase of $10^\circ F$ over those 5 hours.

The increase wasn't constant, however. The temperature rose from 9 a.m. and 10 a.m., stayed steady for an hour, then rose again.

- On average, how fast was the temperature rising between 9 a.m. and 2 p.m.?

  Let's calculate the **average rate of change** and measure the temperature change per hour. We do that by finding the difference in the temperature between 9 a.m. and 2 p.m. and dividing it by the number of hours in that interval.

$$\text{average rate of change} = \frac{45-35}{5} = \frac{10}{5} = 2$$

  On average, the temperature between 9 a.m. and 2 p.m. increased $2^\circ F$ per hour.

- How quickly was the temperature falling between 2 p.m. and 8 p.m.?

$$\text{average rate of change} = \frac{30-45}{6} = \frac{-15}{6} = -2.5$$

  On average, the temperature between 2 p.m. and 8 p.m. dropped by $2.5^\circ F$ per hour.

In general, we can calculate the average rate of change of a function f, between input values a and b, by dividing the difference in the outputs by the difference in the inputs.

$$\text{average rate of change} = \frac{f(b)-f(a)}{b-a}$$

If the two points on the graph of the function are $(a, f(a))$ and $(b, f(b))$, the average rate of change is the slope of the line that connects the two points.