## Lesson 8: Equations and Graphs

* Let’s write an equation for a parabola.

### 8.1: Focus on Distance

The image shows a parabola with focus $\left(-2,2\right)$ and directrix $y=0$ (the $x$-axis). Points $A$, $B$, and $C$ are on the parabola.



Without using the Pythagorean Theorem, find the distance from each plotted point to the parabola’s focus. Explain your reasoning.

### 8.2: Building an Equation for a Parabola

The image shows a parabola with focus $\left(3,2\right)$ and directrix $y=0$ (the $x$-axis).



1. Write an equation that would allow you to test whether a particular point $\left(x,y\right)$ is on the parabola.
2. The equation you wrote defines the parabola, but it’s not in a very easy-to-read form. Rewrite the equation to be in vertex form: $y=a\left(x−h\right)^{2}+k$, where $\left(h,k\right)$ is the vertex.

### 8.3: Card Sort: Parabolas

Your teacher will give you a set of cards with graphs and equations of parabolas. Match each graph with the equation that represents it.

#### Are you ready for more?

In this section, you have examined points that are equidistant from a given point and a given line. Now consider a set of points that are half as far from a point as they are from a line.

1. Write an equation that describes the set of all points that are $\frac{1}{2}$ as far from the point $\left(5,3\right)$ as they are from the $x$-axis.
2. Use technology to graph your equation. Sketch the graph and describe what it looks like.

### Lesson 8 Summary

The parabola in the image consists of all the points that are the same distance from the point $\left(1,4\right)$ as they are from the line $y=0$. Suppose we want to write an equation for the parabola—that is, an equation that says a given point $\left(x,y\right)$ is on the curve. We can draw a right triangle whose hypotenuse is the distance between point $\left(x,y\right)$ and the focus, $\left(1,4\right)$.



The distance from $\left(x,y\right)$ to the directrix, or the line $y=0$, is $y$ units. By definition, the distance from $\left(x,y\right)$ to the focus must be equal to the distance from the point to the directrix. So, the distance from $\left(x,y\right)$ to the focus can be labeled with $y$. To find the lengths of the legs of the right triangle, subtract the corresponding coordinates of the point $\left(x,y\right)$ and the focus, $\left(1,4\right)$. Substitute the expressions for the side lengths into the Pythagorean Theorem to get an equation defining the parabola.

$\left(x−1\right)^{2}+\left(y−4\right)^{2}=y^{2}$

To get the equation looking more familiar, rewrite it in vertex form, or $y=a\left(x−h\right)^{2}+k$ where $\left(h,k\right)$ is the vertex.

$\left(x−1\right)^{2}+\left(y−4\right)^{2}=y^{2}$

$\left(x−1\right)^{2}+y^{2}−8y+16=y^{2}$

$\left(x−1\right)^{2}−8y+16=0$

$-8y=-\left(x−1\right)^{2}−16$

$y=\frac{1}{8}\left(x−1\right)^{2}+2$



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