

# Lesson 17: Lines in Triangles

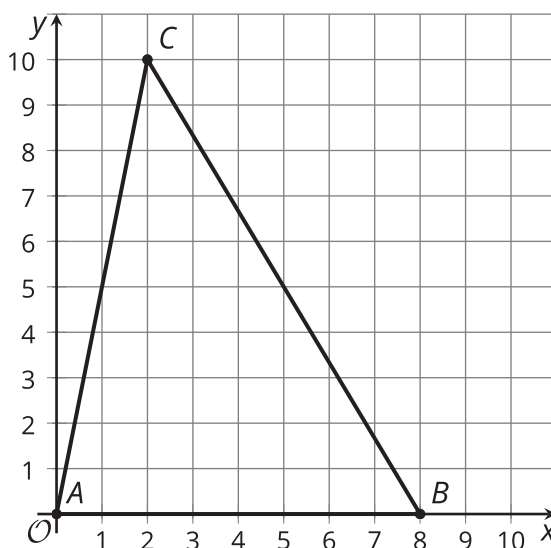
- Let's investigate more special segments in triangles.

## 17.1: Folding Altitudes

Draw a triangle on tracing paper. Fold the altitude from each vertex.

## 17.2: Altitude Attributes

Triangle  $ABC$  is graphed.



1. Find the slope of each side of the triangle.
2. Find the slope of each altitude of the triangle.
3. Sketch the altitudes. Label the point of intersection  $H$ .

4. Write equations for all 3 altitudes.

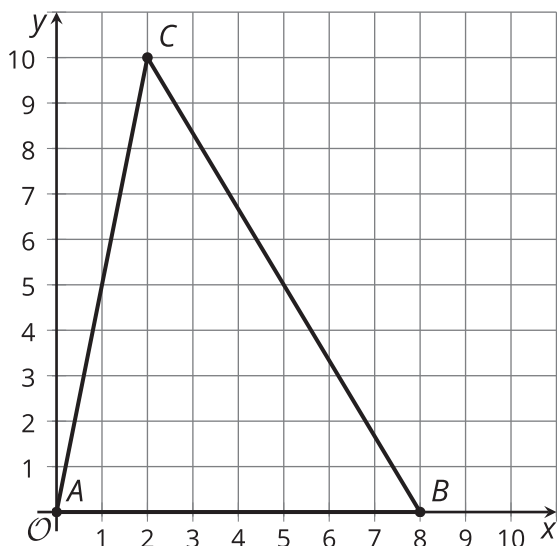
5. Use the equations to find the coordinates of  $H$  and verify algebraically that the altitudes all intersect at  $H$ .

### Are you ready for more?

Any triangle  $ABC$  can be translated, rotated, and dilated so that the image  $A'$  lies on the origin,  $B'$  lies on the point  $(1, 0)$ , and  $C'$  has position  $(a, b)$ . Use this as a starting point to prove that the altitudes of all triangles all meet at the same point.

## 17.3: Percolating on Perpendicular Bisectors

Triangle  $ABC$  is graphed.



1. Find the midpoint of each side of the triangle.
2. Sketch the perpendicular bisectors, using an index card to help draw 90 degree angles. Label the intersection point  $P$ .
3. Write equations for all 3 perpendicular bisectors.
  
4. Use the equations to find the coordinates of  $P$  and verify algebraically that the perpendicular bisectors all intersect at  $P$ .

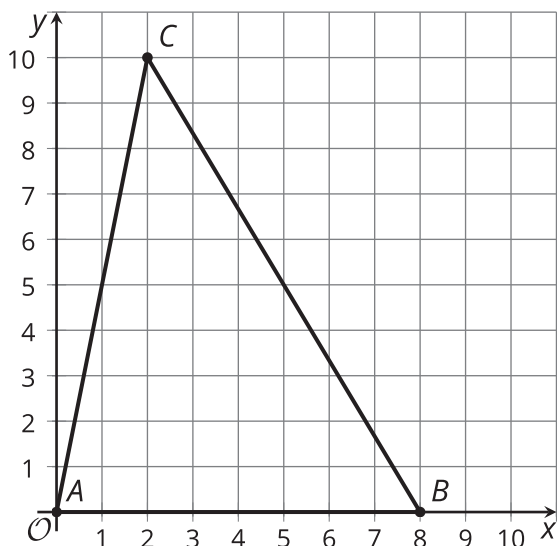
## 17.4: Perks of Perpendicular Bisectors

Consider triangle  $ABC$  from an earlier activity.

1. What is the distance from  $A$  to  $P$ , the intersection point of the perpendicular bisectors of the triangle's sides? Round to the nearest tenth.
2. Write the equation of a circle with center  $P$  and radius  $AP$ .
3. Construct the circle. What do you notice?
4. Verify your hypothesis algebraically.

## 17.5: Amazing Points

Consider triangle  $ABC$  from earlier activities.



1. Plot point  $H$ , the intersection point of the altitudes.
2. Plot point  $P$ , the intersection point of the perpendicular bisectors.
3. Find the point where the 3 medians of the triangle intersect. Plot this point and label it  $J$ .
  
4. What seems to be true about points  $H$ ,  $P$ , and  $J$ ? Prove that your observation is true.

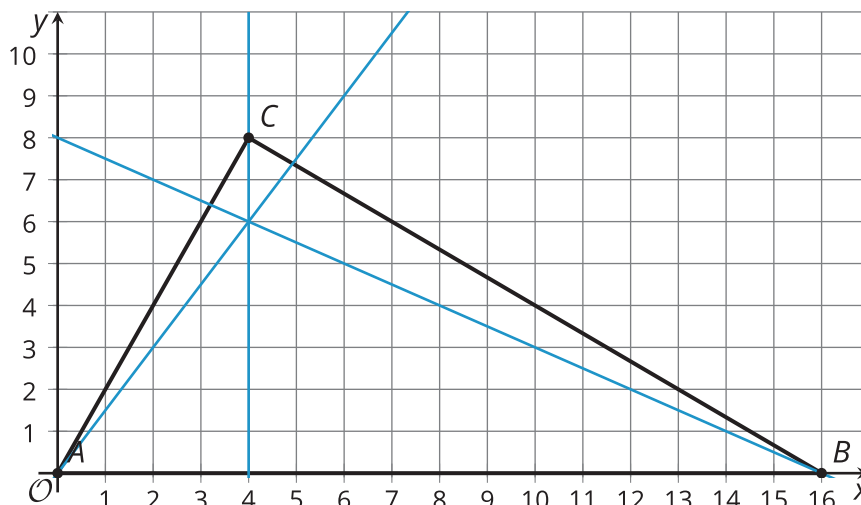
## 17.6: Tiling the (Coordinate) Plane

A tessellation covers the entire plane with shapes that do not overlap or leave gaps.

1. Tile the plane with congruent rectangles:
  - a. Draw the rectangles on your grid.
  - b. Write the equations for lines that outline 1 rectangle.
  
  
  
  
  
  
  
  
  
  
2. Tile the plane with congruent right triangles:
  - a. Draw the right triangles on your grid.
  - b. Write the equations for lines that outline 1 right triangle.
  
  
  
  
  
  
  
  
  
  
3. Tile the plane with any other shapes:
  - a. Draw the shapes on your grid.
  - b. Write the equations for lines that outline 1 of the shapes.

## Lesson 17 Summary

The 3 medians of a triangle always intersect in 1 point. We can use coordinate geometry to show that the altitudes of a triangle intersect in 1 point, too. The 3 altitudes of triangle  $ABC$  are shown here. They appear to intersect at the point  $(4, 6)$ . By finding their equations, we can prove this is true.



The slopes of sides  $AB$ ,  $BC$ , and  $AC$  are  $0$ ,  $-\frac{2}{3}$ , and  $2$ . The altitude from  $C$  is the vertical line  $x = 4$ . The slope of the altitude from  $A$  is  $\frac{3}{2}$ . Since the altitude goes through  $(0, 0)$ , its equation is  $y = \frac{3}{2}x$ . The slope of the altitude from  $B$  is  $-\frac{1}{2}$ . Following this slope over to the  $y$ -axis we can see that the  $y$ -intercept is  $8$ . So the equation for this altitude is  $y = -\frac{1}{2}x + 8$ .

We can now verify that  $(4, 6)$  lies on all 3 altitudes by showing that the point satisfies the 3 equations. By substitution we see that each equation is true when  $x = 4$  and  $y = 6$ .