Unit 7 Lesson 21: Sums and Products of Rational and Irrational Numbers

1 Operations on Integers (Warm up)

Student Task Statement

Here are some examples of integers:

-25   -10   -2   -1   0   5   9   40

1. Experiment with adding any two numbers from the list (or other integers of your choice). Try to find one or more examples of two integers that:

   a. add up to another integer

   b. add up to a number that is *not* an integer

2. Experiment with multiplying any two numbers from the list (or other integers of your choice). Try to find one or more examples of two integers that:

   a. multiply to make another integer

   b. multiply to make a number that is *not* an integer

2 Sums and Products of Rational Numbers

Student Task Statement

1. Here are a few examples of adding two rational numbers. Is each sum a rational number? Be prepared to explain how you know.

   a. $4 + 0.175 = 4.175$

   b. $\frac{1}{2} + \frac{4}{5} = \frac{5}{10} + \frac{8}{10} = \frac{13}{10}$

   c. $-0.75 + \frac{14}{8} = \frac{-6}{8} + \frac{14}{8} = \frac{8}{8} = 1$

   d. a is an integer: $\frac{2}{3} + \frac{a}{15} = \frac{10}{15} + \frac{a}{15} = \frac{10+a}{15}$

2. Here is a way to explain why the sum of two rational numbers is rational.

Suppose $\frac{a}{b}$ and $\frac{c}{d}$ are fractions. That means that a, b, c, and d are integers, and b and d are not 0.

   a. Find the sum of $\frac{a}{b}$ and $\frac{c}{d}$. Show your reasoning.

   b. In the sum, are the numerator and the denominator integers? How do you know?

   c. Use your responses to explain why the sum of $\frac{a}{b} + \frac{c}{d}$ is a rational number.

3. Use the same reasoning as in the previous question to explain why the product of two rational numbers, $\frac{a}{b} \cdot \frac{c}{d}$, must be rational.

3 Sums and Products of Rational and Irrational Numbers

Student Task Statement

1. Here is a way to explain why $\sqrt{2} + \frac{1}{9}$ is irrational.

   ○ Let s be the sum of $\sqrt{2}$ and $\frac{1}{9}$, or $s = \sqrt{2} + \frac{1}{9}$.

   ○ Suppose s is rational.

   a. Would $s + \text{-}\frac{1}{9}$ be rational or irrational? Explain how you know.

   b. Evaluate $s + \text{-}\frac{1}{9}$. Is the sum rational or irrational?

   c. Use your responses so far to explain why s cannot be a rational number, and therefore $\sqrt{2} + \frac{1}{9}$ cannot be rational.

2. Use the same reasoning as in the earlier question to explain why $\sqrt{2} \cdot \frac{1}{9}$ is irrational.

4 Equations with Different Kinds of Solutions (Optional)

Student Task Statement

1. Consider the equation $4x^2 + bx + 9 = 0$. Find a value of b so that the equation has:

    a. 2 rational solutions

    b. 2 irrational solutions

    c. 1 solution

    d. no solutions

2. Describe all the values of b that produce 2, 1, and no solutions.

3. Write a new quadratic equation with each type of solution. Be prepared to explain how you know that your equation has the specified type and number of solutions.

    a. no solutions

    b. 2 irrational solutions

    c. 2 rational solutions

    d. 1 solution