Unit 7 Lesson 19: Deriving the Quadratic Formula

1 Studying Structure (Warm up)

Student Task Statement

Here are some perfect squares in factored and standard forms, and an expression showing how the two forms are related.

1. Study the first few examples, and then complete the missing numbers in the rest of the table.

factored form		standard form
$(x+4)^2$	$(1x)^2 + 2(\underline{}x)(\underline{}) + 4^2$	$x^2 + 8x + 16$
$(2x+5)^2$	$(2x)^2 + 2(\underline{}x)(\underline{}) + 5^2$	$4x^2 + 20x + 25$
$(3x-4)^2$	$(3x)^2 + 2(\underline{}x)(\underline{}) + (\underline{})^2$	$9x^2 - 24x + 16$
$(5x+\underline{})^2$	$(\underline{}x)^2 + 2(\underline{}x)(\underline{}) + (\underline{})^2$	$25x^2 + 30x + \underline{}$
$(kx+m)^2$	$(\underline{}x)^2 + 2(\underline{}x)(\underline{}) + (\underline{})^2$	$\underline{}x^2 + \underline{}x + \underline{}$

2. Look at the expression in the last row of the table. If $ax^2 + bx + c$ is equivalent to $(kx+m)^2$, how are a, b, and c related to k and m?

2 Complete the Square using a Placeholder

Student Task Statement

1. One way to solve the quadratic equation $x^2 + 5x + 3 = 0$ is by completing the square. A partially solved equation is shown here. Study the steps.

   Then, knowing that P is a placeholder for $2x$, continue to solve for x but without evaluating any part of the expression. Be prepared to explain each step.

$x^2 + 5x + 3 = 0$	Original equation
$4x^2 + 20x + 12 = 0$	Multiply each side by 4
$4x^2 + 20x = \text{-}12$	Subtract 12 from each side
$(2x)^2 + 10(2x) = \text{-}12$	Rewrite $4x^2$ as $(2x)^2$ and $20x$ as $10(2x)$
$P^2 + 10P = \text{-}12$	Use P as a placeholder for $2x$

   $$P^2 + 10P + \underline{\quad}^2 = \text{-}12 + \underline{\quad}^2$$

   $$(P + \underline{\quad})^2 = \text{-}12 + \underline{\quad}^2$$

   $$P + \underline{\quad} = \pm\sqrt{\text{-}12 + \underline{\quad}^2}$$

   $$P = \underline{\quad} \pm \sqrt{\text{-}12 + \underline{\quad}^2}$$

   $$P = \underline{\quad} \pm \sqrt{\underline{\quad}^2 - 12}$$

   $$2x = \underline{\quad} \pm \sqrt{\underline{\quad}^2 - 12}$$

   $$x =$$

2. Explain how the solution is related to the quadratic formula.

3 Decoding the Quadratic Formula

Student Task Statement

Here is one way to make sense of how the quadratic formula came about. Study the derivation until you can explain what happened in each step. Record your explanation next to each step.

$$ax^2 + bx + c = 0$$

$$4a^2x^2 + 4abx + 4ac = 0$$

$$4a^2x^2 + 4abx = \text{-}4ac$$

$$(2ax)^2 + 2b(2ax) = \text{-}4ac$$

$$M^2 + 2bM = \text{-}4ac$$

$$M^2 + 2bM + b^2 = \text{-}4ac + b^2$$

$$(M + b)^2 = b^2 - 4ac$$

$$M + b = \pm\sqrt{b^2 - 4ac}$$

$$M = \text{-}b \pm \sqrt{b^2 - 4ac}$$

$$2ax = \text{-}b \pm \sqrt{b^2 - 4ac}$$

$$x = \frac{\text{-}b \pm \sqrt{b^2 - 4ac}}{2a}$$