

Lesson 1: A Towering Sequence

- Let's explore the Tower of Hanoi.

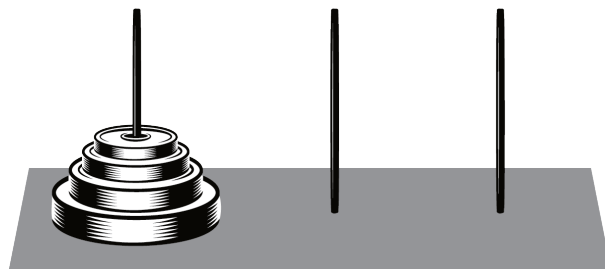
1.1: What's Next?

Here is a rule for making a list of numbers: *Each number is 1 less than twice the previous number.*

Pick a number to start with, then follow the rule to build a list of 5 numbers.

1.2: The Tower of Hanoi

In the Tower of Hanoi puzzle, a set of discs sits on a peg, while there are 2 other empty pegs.



A *move* in the Tower of Hanoi puzzle involves taking a disc and moving it to another peg. There are two rules:

- Only move 1 disc at a time.
- Never put a larger disc on top of a smaller one.

You complete the puzzle by building the complete tower on a different peg than the starting peg.

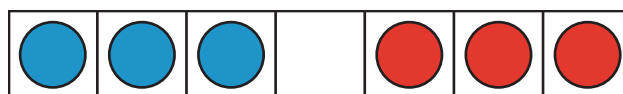
1. Using 3 discs, complete the puzzle. What is the smallest number of moves you can find?
2. Using 4 discs, complete the puzzle. What is the smallest number of moves you can find?
3. Jada says she used the solution for 3 discs to help her solve the puzzle for 4 discs. Describe how this might happen.
4. How many moves do you think it will take to complete a puzzle with 5 discs? Explain or show your reasoning.
5. How many moves do you think it will take to complete a puzzle with 7 discs?

Are you ready for more?

A legend says that a Tower of Hanoi puzzle with 64 discs is being solved, one move per second. How long will it take to solve this puzzle? Explain how you know.

1.3: Checker Jumping Puzzle

Some checkers are lined up, with blue on one side, red on the other, with one empty space between them. A *move* in this checker game pushes any checker forward 1 space, or jumps over any 1 checker of the other color. Jumping the same color is not allowed, moving backwards is not allowed, and 2 checkers cannot occupy the same space.



You complete the puzzle by switching the colors completely: ending up with blue on the right, red on the left, with 1 empty space between them.

1. Using 1 checker on each side, complete the puzzle. What is the smallest number of moves needed?
2. Using 3 checkers on each side, complete the puzzle. What is the smallest number of moves needed?
3. Estimate the number of moves needed if there are 2 or 4 checkers on each side, then test your guesses.
4. Noah says he used the solution for 3 checkers on each side to help him solve the puzzle for 4 checkers. Describe how this might happen.
5. How many moves do you think it will take to complete a puzzle with 7 checkers on each side?

Lesson 1 Summary

A list of numbers like 3, 5, 7, 9, 11, . . . or 1, 5, 13, 29, 61, . . . is called a **sequence**.

There are many ways to define a sequence, but one way is to describe how each **term** relates to the one before it. For example, the sequence 3, 5, 7, 9, 11, . . . can be described this way: the starting term is 3, then each following term is 2 more than the one before it. The sequence 1, 5, 13, 29, 61, . . . can be described as: the starting term is 1, then each following term is the sum of 3 and twice the previous term.

Throughout this unit, we will study several types of sequences along with ways to represent them.