## Lesson 8: Unknown Exponents

* Let’s find unknown exponents.

### 8.1: A Bunch of $x$’s

Solve each equation. Be prepared to explain your reasoning.

1. $\frac{x}{3}=12$
2. $3x^{2}=12$
3. $x^{3}=12$
4. $\sqrt[3]{x}=12$
5. $\sqrt{3x}=12$
6. $\frac{3}{x}=12$

### 8.2: A Tessellated Trapezoid

Here is a pattern showing a trapezoid being successively decomposed into four similar trapezoids at each step.



1. If $n$ is the step number, how many of the smallest trapezoids are there when $n$ is 4? What about when $n$ is 10?
2. At a certain step, there are 262,144 smallest trapezoids.
	1. Write an equation to represent the relationship between $n$ and the number of trapezoids in that step.
	2. Explain to a partner how you might find the value of that step number.

### 8.3: Successive Splitting



In a lab, a colony of 100 bacteria is placed on a petri dish. The population triples every hour.

1. How would you estimate or find the population of bacteria in:
	1. 4 hours?
	2. 90 minutes?
	3. $\frac{1}{2}$ hour?
2. How would you estimate or find the number of hours it would take the population to grow to:
	1. 1,000 bacteria?
	2. double the initial population?

#### Are you ready for more?

A $1,000 investment increases in value by 5% each year. About how many years does it take for the value of the investment to double? Explain how you know.

### 8.4: Missing Values

Complete the tables.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $x$ |   |   | -1 | 0 | $\frac{1}{2}$ | 1 |   |   | 5 |   |   |
| $2^{x}$ | $\frac{1}{32}$ | $\frac{1}{4}$ | $\frac{1}{2}$ |   |   |   | 4 | 16 |   | 256 | 1,024 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $x$ |   |   |   | $\frac{1}{3}$ | $\frac{1}{2}$ |   |   |   |   |
| $5^{x}$ | $\frac{1}{25}$ | $\frac{1}{5}$ | 1 |   |   | 5 | 125 | 625 | 3,125 |

Be prepared to explain how you found the missing values.

### Lesson 8 Summary

Sometimes we know the value of an exponential expression but we don’t know the exponent that produces that value.

For example, suppose the population of a town was 1 thousand. Since then, the population has doubled every decade and is currently at 32 thousand. How many decades has it been since the population was 1 thousand?

If we say that $d$ is the number of decades since the population was 1 thousand, then $1⋅2^{d}$, or just $2^{d}$, represents the population, in thousands, after $d$ decades. To answer the question, we need to find the exponent in $2^{d}=32$. We can reason that since $2^{5}=32$, it has been 5 decades since the population was 1 thousand people.

When did the town have 250 people? Assuming that the doubling started before the population was measured to be 1 thousand, we can write: $2^{d}=0.25$ or $2^{d}=\frac{1}{4}$. We know that $2^{-2}=\frac{1}{4}$, so the exponent $d$ has a value of -2. The population was 250 two decades before it was 1,000.

But it may not always be so straightforward to calculate. For example, it is harder to tell the value of $d$ in $2^{d}=805$ or in $2^{d}=4.5$. In upcoming lessons, we’ll learn more ways to find unknown exponents.



© CC BY 2019 by Illustrative Mathematics®