

Lesson 8: Combining Bases

Let's multiply expressions with different bases.

8.1: Same Exponent, Different Base

1. Evaluate $5^3 \cdot 2^3$
2. Evaluate 10^3

8.2: Power of Products

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the "expanded" column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$		21^2
$2^4 \cdot 3^4$		
		15^3
		30^4
$2^4 \cdot x^4$		
$a^n \cdot b^n$		
$7^4 \cdot 2^4 \cdot 5^4$		

2. Can you write $2^3 \cdot 3^4$ with a single exponent? What happens if neither the exponents nor the bases are the same? Explain or show your reasoning.

8.3: How Many Ways Can You Make 3,600?

Your teacher will give your group tools for creating a visual display to play a game. Divide the display into 3 columns, with these headers:

$$a^n \cdot a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$a^n \cdot b^n = (a \cdot b)^n$$

How to play:

When the time starts, you and your group will write as many expressions as you can that equal a specific number using one of the exponent rules on your board. When the time is up, compare your expressions with another group to see how many points you earn.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rule you claimed.
- If an expression uses negative exponents, you get 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the three exponent rules.

Are you ready for more?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one *more* than a multiple of 3, or one *less* than a multiple of 3.

1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.
2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.
3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.

Lesson 8 Summary

Before this lesson, we made rules for multiplying and dividing expressions with exponents that only work when the expressions have the *same* base. For example,

$$10^3 \cdot 10^2 = 10^5$$

or

$$2^6 \div 2^2 = 2^4$$

In this lesson, we studied how to combine expressions with the same exponent, but *different* bases. For example, we can write $2^3 \cdot 5^3$ as $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$. Regrouping this as $(2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5)$ shows that

$$\begin{aligned} 2^3 \cdot 5^3 &= (2 \cdot 5)^3 \\ &= 10^3 \end{aligned}$$

Notice that the 2 and 5 in the previous example could be replaced with different numbers or even variables. For example, if a and b are variables then $a^3 \cdot b^3 = (a \cdot b)^3$. More generally, for a positive number n ,

$$a^n \cdot b^n = (a \cdot b)^n$$

because both sides have exactly n factors that are a and n factors that are b .