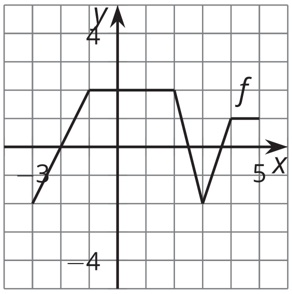
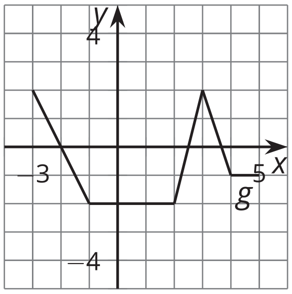
Lesson 4: Reflecting Functions

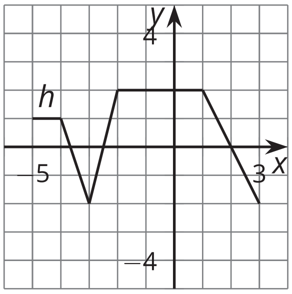
- Let’s reflect some graphs.

4.1: Notice and Wonder: Reflections

What do you notice? What do you wonder?

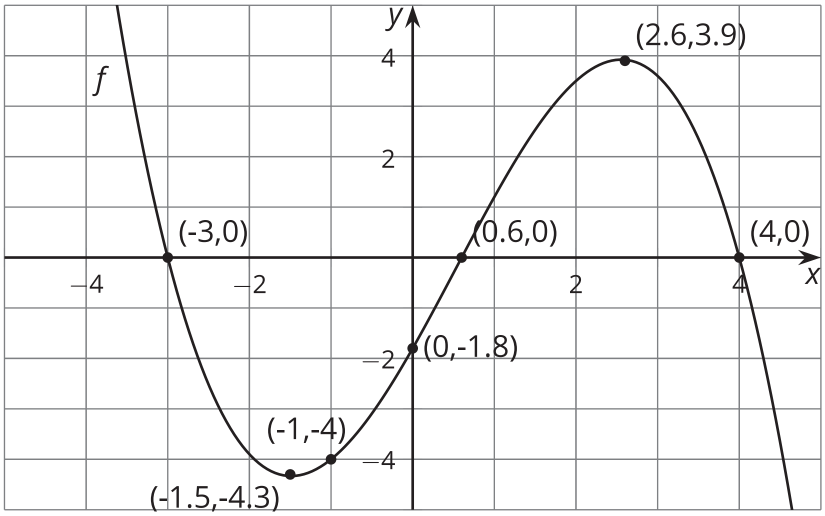






4.2: Reflecting Across

Here is the graph of function f and a table of values.

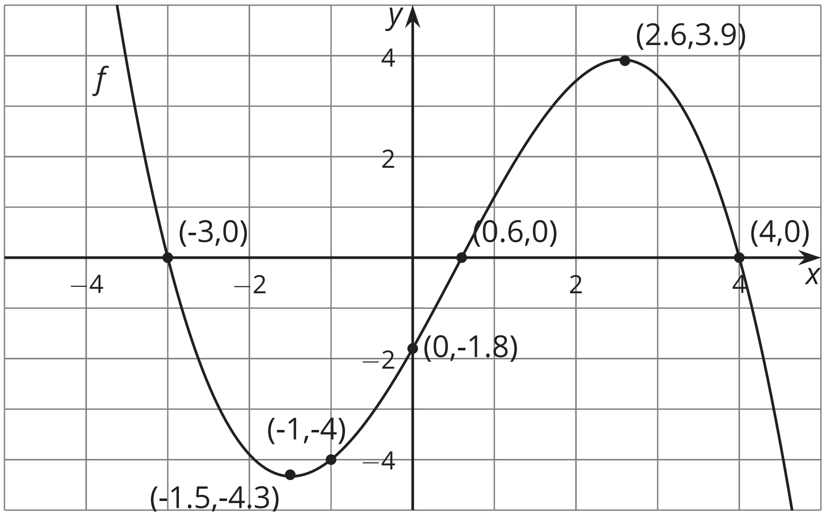


x	$f(x)$	$g(x) = -f(x)$
-3	0	
-1.5	-4.3	
-1	-4	
0	-1.8	
0.6	0	
2.6	3.9	
4	0	

1. Let g be the function defined by $g(x) = -f(x)$. Complete the table.
2. Sketch the graph of g on the same axes as the graph of f but in a different color.
3. Describe how to transform the graph of f into the graph of g. Explain how the equation produces this transformation.

4.3: Reflecting Across a Different Way

Here is another copy of the graph of f from the earlier activity. This time, let h be the function defined by $h(x) = f(-x)$.



1. Use the definition of h to find $h(0)$. Does your answer agree with your prediction?

2. What does your prediction tell you about $h(-0.6)$? Does your answer agree with the definition of h?

3. Complete the tables. The values for x will not be the same for the two tables.

x	$f(x)$
-3	0
-1.5	-4.3
-1	-4
0	-1.8
0.6	0
2.6	3.9
4	0

x	$h(x) = f(-x)$

4. Sketch the graph of h on the same axes as the graph of f but in a different color.

5. Describe what happened to the graph of f to transform it into the graph of h. Explain how the equation produces this transformation.

Are you ready for more?

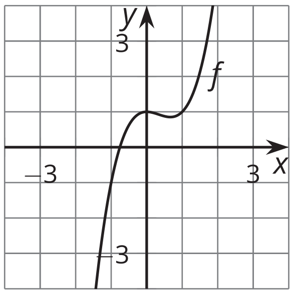
1. Describe how the graph of h relates to the graph of g defined in the earlier activity.

2. Write an equation relating h and g.

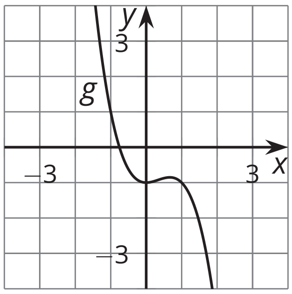
Lesson 4 Summary

Here are graphs of the functions f, g, and h, where $g(x) = \text{-}f(x)$ and $h(x) = f(\text{-}x)$. How do these equations match the transformation we see from f to g and from f to h?

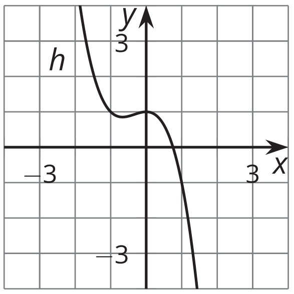
$f(x)$



$g(x) = \text{-}f(x)$



$h(x) = f(\text{-}x)$



Considering first the equation $g(x) = \text{-}f(x)$, we know that for the same input x, the value of $g(x)$ will be the opposite of the value of $f(x)$. For example, since $f(0) = 1$, we know that $g(0) = \text{-}f(0) = \text{-}1$. We can see this relationship in the graphs where g is the reflection of f across the x-axis.

Looking at $h(x) = f(\text{-}x)$, this equation tells us that the two functions have the same output for opposite inputs. For example, 1 and -1 are opposites, so $h(1) = f(\text{-}1)$ (and $h(\text{-}1) = f(1)$ is also true!). We can see this relationship in the graphs where h is the reflection of f across the y-axis.