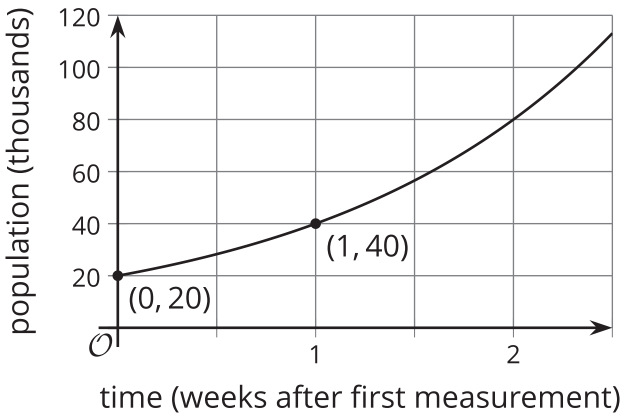
Family Support Materials

Exponential Functions and Equations

In this unit, your student will look at exponential functions and use them to solve problems. Exponential functions are used to model many real-world situations. For example,

- Many populations grow exponentially, especially when resources are readily available.
- Contagious diseases can spread exponentially when first introduced to a population.
- Radioactive substances, like those used in medical treatments or nuclear power plants, decay or decrease exponentially in predictable ways.

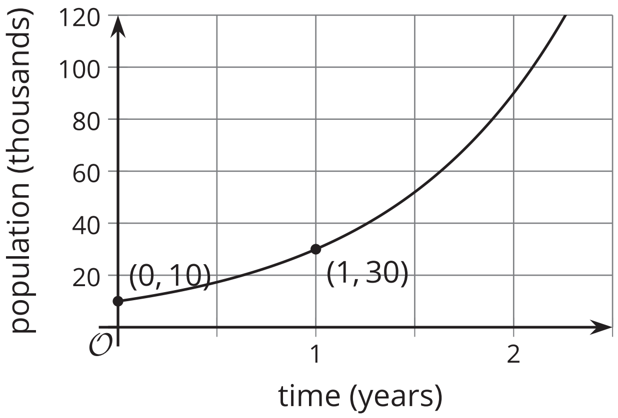
Here is a graph showing an insect population p, in thousands, w weeks after it was first measured.



The population is growing exponentially, doubling each week. An equation relating p and w is $p = 20 \cdot 2^w$. But what if we want to see how quickly the insect population grows each day? Because the growth is exponential, we know it grows by the same factor each day. If one week of growth means multiplying by 2, then one day of growth means multiplying by the seventh root of 2, $2^{\frac{1}{7}}$, since this is the number whose seventh power is 2. Using this factor, if d is the number of days since the insect population was measured, the relationship between p and d is $p = 20 \cdot \left(2^{\frac{1}{7}}\right)^d$. Now we have an equation we can use to estimate the population by days instead of by weeks.

Here is a task to try with your student:

Here is the graph of a different exponentially increasing population a, in thousands, given by the equation $a = 10 \cdot 3^t$. Here t is time measured in years.



1. What do the labeled points $(0, 10)$ and $(1, 30)$ mean in this situation?

2. By what factor does the population grow each month? Hint: how can you use the number of months in a year to express this factor?

3. Write an equation for the population, in thousands, m months after it was first measured.

4. After about how many months did the population reach 50,000?

Solution:

1. The point $(0, 10)$ means that the population was 10,000 when first measured and was 30,000 after 1 year.

2. $3^{\frac{1}{12}}$

3. $p = 10 \cdot \left(3^{\frac{1}{12}}\right)^m$

4. between 17 and 18 months