## Lesson 4: Proportional Relationships and Equations

Let's write equations describing proportional relationships.

## 4.1: Number Talk: Division

Find each quotient mentally.
$645 \div 100$
$645 \div 50$
$48.6 \div 30$
$48.6 \div x$

## 4.2: Feeding a Crowd, Revisited

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.
a. How many people will 1 cup of rice serve?
b. How many people will 3 cups of rice serve?
12 cups? 43 cups?
c. How many people will $x$ cups of rice serve?

| cups of dry rice | number of people |
| :---: | :---: |
| 1 |  |
| 2 | 6 |
| 3 |  |
| 12 |  |
| 43 |  |
| $x$ |  |

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.
a. How many people will 1 spring roll serve?
b. How many people will 10 spring rolls serve? 16 spring rolls? 25 spring rolls?
c. How many people will $n$ spring rolls serve?

| number of spring rolls | number of people |
| :---: | :---: |
| 1 |  |
| 6 | 3 |
| 10 |  |
| 16 |  |
| 25 |  |
| $n$ |  |

3. How was completing this table different from the previous table? How was it the same?

## 4.3: Denver to Chicago

A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.


1. Complete the table.

| time (hours) | distance (miles) | speed (miles per hour) |
| :---: | :---: | :---: |
| 1 |  |  |
| 1.5 | 915 |  |
| 2 |  |  |
| 2.5 |  |  |
| $t$ |  |  |

2. How far does the plane fly in one hour?
3. How far would the plane fly in $t$ hours at this speed?
4. If $d$ represents the distance that the plane flies at this speed for $t$ hours, write an equation that relates $t$ and $d$.
5. How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

## Are you ready for more?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1. How long does it take light from Proxima Centauri to reach Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)
2. There are two twins. One twin leaves on a spaceship to explore the planet near Proxima Centauri traveling at $90 \%$ of the speed of light, while the other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri? (Do you think the answer would be the same for the other twin? Consider researching "The Twin Paradox" to learn more.)

## 4.4: Revisiting Bread Dough

A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of honey to flour.

1. Complete the table.
2. If $f$ is the cups of flour needed for $h$ tablespoons of honey, write an equation that relates $f$ and $h$.
3. How much flour is needed for 15 tablespoons of honey? 17 tablespoons? Explain or show your reasoning.

| honey (tbsp) | flour (c) |
| :---: | :---: |
| 1 |  |
| 8 | 10 |
| 16 |  |
| 20 |  |
| $h$ |  |

## Lesson 4 Summary

The table shows the amount of red paint and blue paint needed to make a certain shade of purple paint, called Venusian Sunset.

Note that "parts" can be any unit for volume. If we mix 3 cups of red with 12 cups of blue, you will get the same shade as if we mix 3 teaspoons of red with 12 teaspoons of blue.

| red paint <br> (parts) | blue paint <br> (parts) |
| :---: | :---: |
| 3 | 12 |
| 1 | 4 |
| 7 | 28 |
| $\frac{1}{4}$ | 1 |
| $r$ | $4 r$ |

The last row in the table says that if we know the amount of red paint needed, $r$, we can always multiply it by 4 to find the amount of blue paint needed, $b$, to mix with it to make Venusian Sunset. We can say this more succinctly with the equation $b=4 r$. So the amount of blue paint is proportional to the amount of red paint and the constant of proportionality is 4 .

We can also look at this relationship the other way around.

If we know the amount of blue paint needed, $b$, we can always multiply it by $\frac{1}{4}$ to find the amount of red paint needed, $r$, to mix with it to make Venusian Sunset. So $r=\frac{1}{4} b$. The amount of red paint is proportional to the amount of blue paint and the constant of proportionality $\frac{1}{4}$.

| blue paint <br> (parts) | red paint <br> (parts) |
| :---: | :---: |
| 12 | 3 |
| 4 | 1 |
| 28 | 7 |
| 1 | $\frac{1}{4}$ |
| $b$ | $\frac{1}{4} b$ |

In general, when $y$ is proportional to $x$, we can always multiply $x$ by the same number $k$-the constant of proportionality-to get $y$. We can write this much more succinctly with the equation $y=k x$.

