Lesson 18: Expressed in Different Ways

Let's write exponential expressions in different ways.

18.1: Math Talk: Equal Expressions

Decide if each expression is equal to $(1.21)^{100}$.

$\left((1.21)^{10}\right)^{10}$

$\left((1.21)^{50}\right)^{50}$

$\left((1.1)^{2}\right)^{100}$

$(1.1)^{200}$

18.2: Population Projections

1. From 1790 to 1860, the United States population, in thousands, is modeled by the equation $P = 4{,}000 \cdot (1.031)^t$ where t is the number of years since 1790.
    a. About how many people were living in the U.S. in 1790? What about in 1860? Show your reasoning.

    b. What is the approximate annual percent increase predicted by the model?

    c. What does the model predict for the population in 2017? Is it accurate? Explain.

2. a. What percent increase does the model predict each decade? Explain.

b. Suppose d represents the number of *decades* since 1790. Write an equation for P in terms of d modeling the population in the US (in thousands).

3. a. What percent increase does the model predict each century? Explain.

b. Suppose c represents the number of centuries since 1790. Write an equation for P in terms of c modeling the population in the United States (in thousands).

18.3: Interest Calculations

Here are three expressions and three descriptions. In each case, $1,000 has been put in an interest-bearing bank account. No withdrawals or other deposits (aside from the earned interest) are made for 6 years.

- $1{,}000 \cdot \left(1 + \frac{0.07}{12}\right)^{72}$
- $1{,}000 \cdot \left(1 + \frac{0.07}{2}\right)^{12}$
- $1{,}000 \cdot \left(\left(1 + \frac{0.07}{12}\right)^{12}\right)^{6}$

- 7% annual interest compounded semi-annually
- 7% annual interest compounded monthly
- 7% annual interest compounded every two months

Sort the expressions and descriptions that represent the same amounts of interest into groups. One group contains more than two expressions. One of the descriptions does not have a match. Write an expression that matches it.

Are you ready for more?

Investing \$1,000 at a 5% annual interest rate for 6 years, compounded every two months, yields \$1,348.18. Without doing any calculations, rank these four possible changes in order of the increase in the interest they would yield from the greatest increase to the least increase:

- Increase the starting amount by \$100.
- Increase the interest rate by 1%.
- Let the account increase for one more year.
- Compound the interest every month instead of every two months.

Once you have made your predictions, calculate the value of each option to see if your ranking was correct.

Lesson 18 Summary

Expressions can be written in different ways to highlight different aspects of a situation or to help us better understand what is happening. A **growth rate** tells us the percent change. As always, in percent change situations, it is important to know if the change is an increase or decrease. For example:

- A population is increasing by 20% each year. The growth rate is 20%, so after one year, 0.2 times the population at the beginning of that year is being added. If the initial population is p, the new population is $p + 0.2p$, which equals $(1 + 0.2)p$ or $1.2p$.
- A population is decreasing by 20% each year. The growth rate is -20%, so after one year, 0.2 times the population at the beginning of that year is being lost. If the initial population is p, the new population is $p - 0.2p$, which equals $(1 - 0.2)p$ or $0.8p$.

Suppose the area a covered by a forest is currently 50 square miles and it is growing by 0.2% each year. If t represents time, from now, in years we can express the area of the forest as:

$$a = 50 \cdot (1 + 0.002)^t$$

$$a = 50 \cdot (1.002)^t$$

In this situation, the *growth rate* is 0.002, and the growth factor is 1.002. Because 0.002 is such a small number, however, it maybe difficult to tell from this function how quickly the forest is growing. We may find it more meaningful to measure the growth every decade or every century. There are 10 years in a decade, so to find the growth rate in decades, we can use the expression $(1.002)^{10}$, which is approximately 1.02. This means a growth rate of about 2% per decade. Using d for time, in decades, the area of the forest can be expressed as:

$$a = 50 \cdot \left((1 + 0.002)^{10}\right)^d$$

$$a \approx 50 \cdot (1.02)^d$$

If we measure time in centuries, the growth rate is about 22% per century because $1.002^{100} \approx 1.22$. Using c to measure time, in centuries, our equation for area becomes:

$$a = 50 \cdot \left((1 + 0.002)^{100}\right)^c$$

$$a \approx 50 \cdot (1.22)^c$$