Lesson 9: Side Length Quotients in Similar Triangles

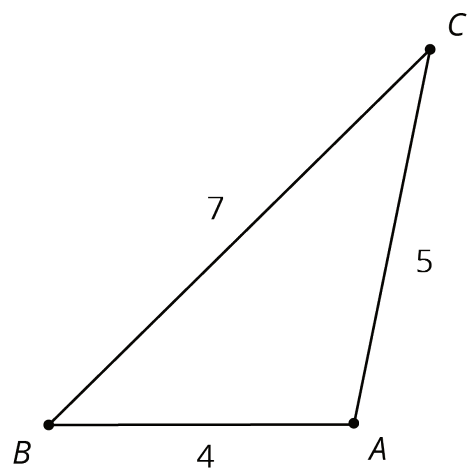
Let's find missing side lengths in triangles.

9.1: Two-three-four and Four-five-six

Triangle A has side lengths 2, 3, and 4. Triangle B has side lengths 4, 5, and 6. Is Triangle A similar to Triangle B?

9.2: Quotients of Sides Within Similar Triangles

Triangle ABC is similar to triangles DEF, GHI, and JKL. The scale factors for the dilations that show triangle ABC is similar to each triangle are in the table.



1. Find the side lengths of triangles DEF, GHI, and JKL. Record them in the table.

triangle	scale factor	length of short side	length of medium side	length of long side
ABC	1	4	5	7
DEF	2			
GHI	3			
JKL	$\frac{1}{2}$			

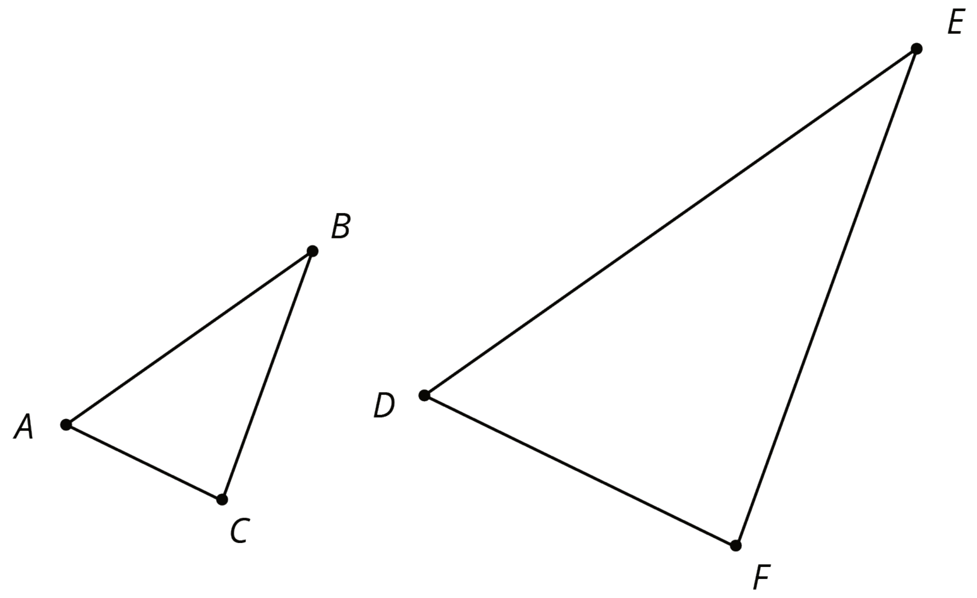
2. Your teacher will assign you one of the three columns. For all four triangles, find the quotient of the triangle side lengths assigned to you and record it in the table. What do you notice about the quotients?

triangle	(long side) ÷ (short side)	(long side) ÷ (medium side)	(medium side) ÷ (short side)
ABC	$\frac{7}{4}$ or 1.75		
DEF			
GHI			
JKL			

3. Compare your results with your partners' and complete your table.

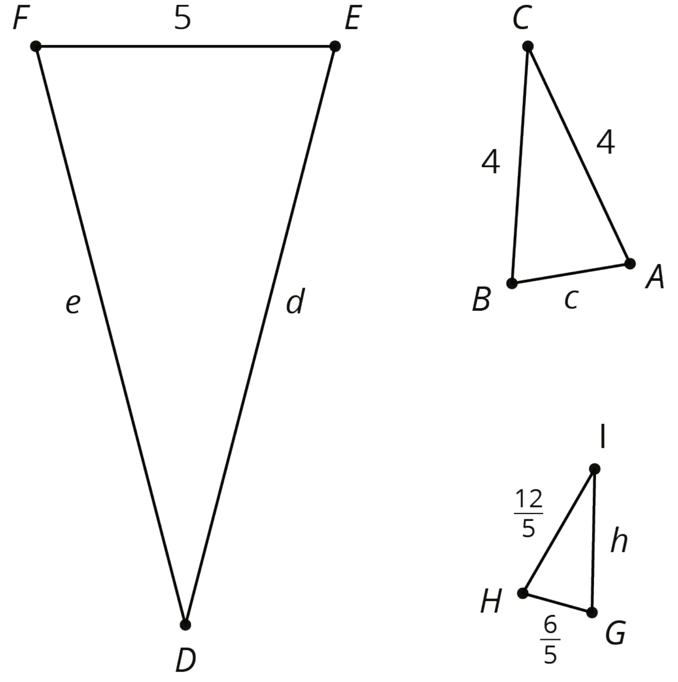
Are you ready for more?

Triangles ABC and DEF are similar. Explain why $\frac{AB}{BC} = \frac{DE}{EF}$.



9.3: Using Side Quotients to Find Side Lengths of Similar Triangles

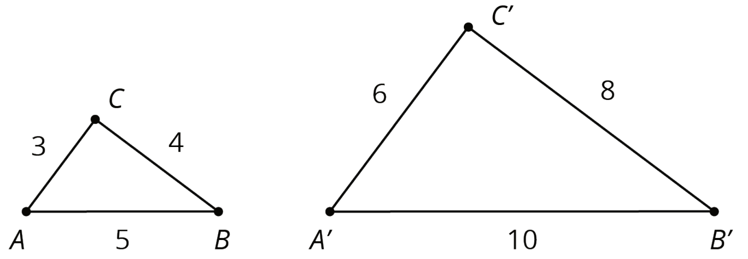
Triangles ABC, EFD, and GHI are all similar. The side lengths of the triangles all have the same units. Find the unknown side lengths.



Lesson 9 Summary

If two polygons are similar, then the side lengths in one polygon are multiplied by the same scale factor to give the corresponding side lengths in the other polygon.

For these triangles the scale factor is 2:



Here is a table that shows relationships between the short and medium length sides of the small and large triangle.

	small triangle	large triangle
medium side	4	8
short side	3	6
(medium side) ÷ (short side)	$\frac{4}{3}$	$\frac{8}{6} = \frac{4}{3}$

The lengths of the medium side and the short side are in a ratio of $4 : 3$. This means that the medium side in each triangle is $\frac{4}{3}$ as long as the short side. This is true for all similar polygons; the ratio between two sides in one polygon is the same as the ratio of the corresponding sides in a similar polygon.

We can use these facts to calculate missing lengths in similar polygons. For example, triangles $A'B'C'$ and ABC shown here are similar. Let's find the length of segment $B'C'$.

In triangle ABC, side BC is twice as long as side AB, so this must be true for any triangle that is similar to triangle ABC. Since $A'B'$ is 1.2 units long and $2 \cdot 1.2 = 2.4$, the length of side $B'C'$ is 2.4 units.