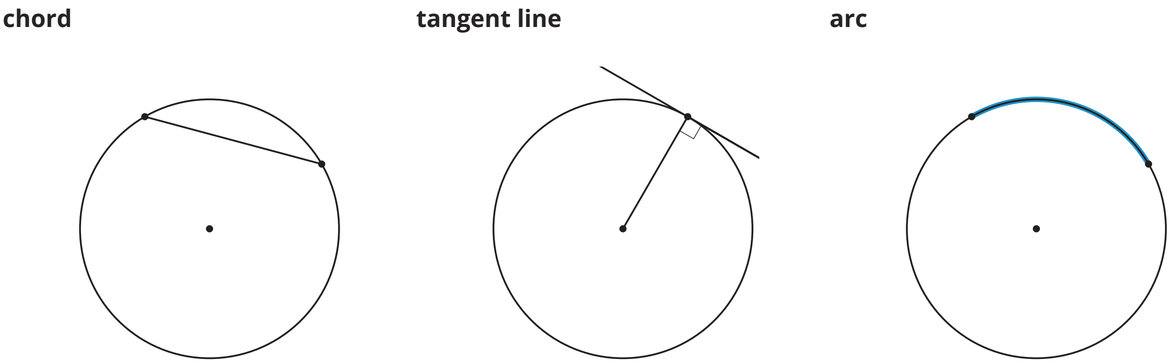
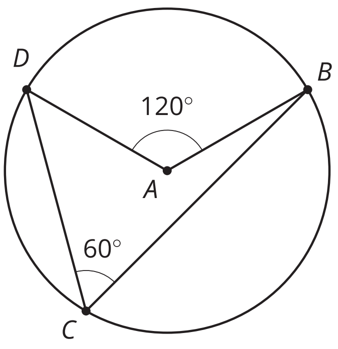
Family Support Materials

Circles

In this unit, your student will study properties of circles. Students start by exploring new vocabulary. In prior units, students worked with radii and diameters of circles. Here, several new concepts are defined: Chords are segments whose endpoints are on a circle. A line tangent to a circle intersects the circle in exactly one point. An arc is a portion of a circle's circumference between 2 endpoints.

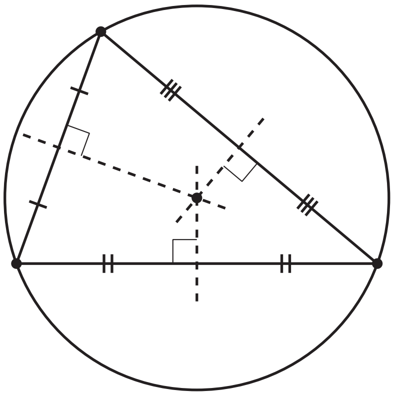


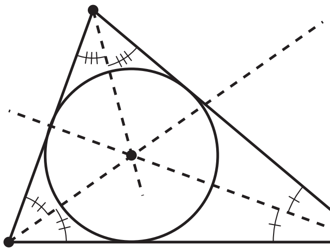
There are also some special angles defined in circles: A central angle is formed by 2 radii, and an inscribed angle is formed by 2 chords that share an endpoint. Your student will identify relationships between chords, tangent lines, arcs, central angles, and inscribed angles. For example, if an inscribed angle and a central angle define the same arc, then the measure of the inscribed angle is half that of the central angle. In the image, angle DCB is an inscribed angle, and its measure is half the measure of the corresponding central angle DAB.



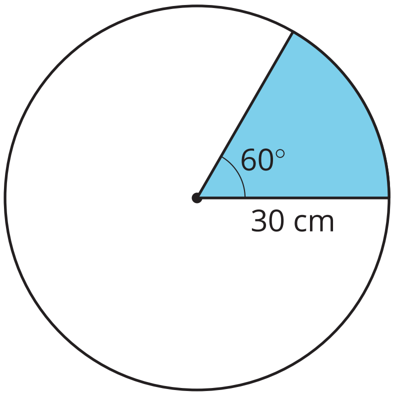
Next, students examine inscribed and circumscribed circles. A circle is said to be circumscribed about a polygon if it passes through each of the polygon's vertices, and it's referred to as an inscribed circle if it is tangent to all sides of the polygon.

All triangles have both circumscribed and inscribed circles. To draw a circumscribed circle for a triangle, construct the perpendicular bisectors of the triangle's sides. These 3 lines meet at a point called the triangle's circumcenter. A circle centered at this point, with radius set to the distance between the circumcenter and a vertex of the triangle, will pass through all the triangle's vertices. To draw a triangle's inscribed circle, construct the triangle's angle bisectors, which all meet at a point called the incenter. The inscribed circle is centered at the incenter, and its radius is the distance from the incenter to any of the triangle’s sides.





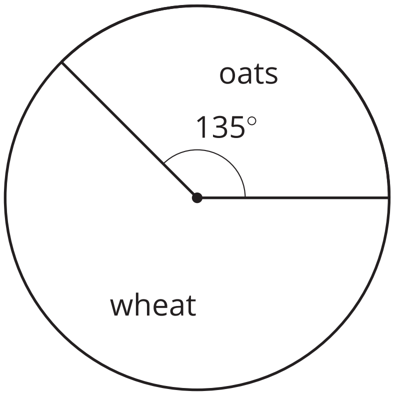
Your student will also study portions of circles. A sector is the region of a circle enclosed between two radii. To find the area of the sector in the image, first calculate the area of the full circle. This area is 900π square centimeters because $\pi(30)^2 = 900\pi$. The sector makes up $\frac{1}{6}$ of the circle because $\frac{60}{360} = \frac{1}{6}$. Multiply this fraction by the total area to find that the area of the sector is 150π square centimeters.



Finally, students have previously measured angles using degrees, but here they learn a new way to measure angles. The radian measure of an angle whose vertex is at the center of a circle is the ratio of the length of the arc defined by the angle to the radius of the circle. That is, $\theta = \frac{\text{arc length}}{\text{radius}}$. Radian measure will be useful when students study trigonometry in future courses.

Here is a task to try with your student:

A farmer has a circular field, created by a watering system that rotates around a center pivot point. The field's radius measures 400 meters. As shown in the image, part of the field is planted with oats and part is planted with wheat.



1. Find the area of the field that is planted with oats.

2. A road runs around the circumference of the circle. Find the length of the arc of the road defined by the wheat part of the field.

Solution:

1. The total area of the field is 160,000π square meters because $\pi(400)^2 = 160{,}000\pi$. The 135 degree sector represents $\frac{3}{8}$ of the field because $\frac{135}{360} = \frac{3}{8}$. Multiply 160,000$\pi$ by $\frac{3}{8}$ to find an area of 60,000π square meters of oats.

2. The total circumference of the field is 800π meters because $2 \cdot \pi \cdot 400 = 800\pi$. The wheat sector takes up $\frac{5}{8}$ of the field because $1 - \frac{3}{8} = \frac{5}{8}$. Multiply 800π by $\frac{5}{8}$ to find that this portion of the road is 500π or about 1,571 meters long.