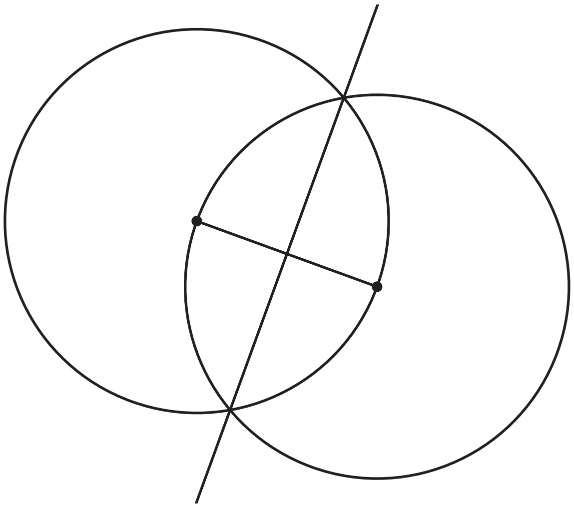
Lesson 14: Bisect It

- Let's prove that some constructions we conjectured about really work.

14.1: Why Does This Construction Work?



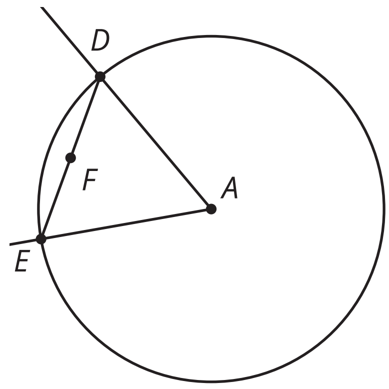
If you are Partner A, explain to your partner what steps were taken to construct the perpendicular bisector in this image.

If you are Partner B, listen to your partner's explanation, and then explain to your partner why these steps produce a line with the properties of a perpendicular bisector.

Then, work together to make sure the main steps in Partner A's explanation have a reason from Partner B's explanation.

14.2: Construction from Definition

Han, Clare, and Andre thought of a way to construct an angle bisector. They used a circle to construct points D and E the same distance from A. Then they connected D and E and found the midpoint of segment DE. They thought that ray AF would be the bisector of angle DAE. Mark the given information on the diagram:



Han's rough-draft justification: F is the midpoint of segment DE. I noticed that F is also on the perpendicular bisector of angle DAE.

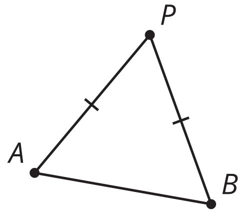
Clare's rough-draft justification: Since segment DA is congruent to segment EA, triangle DEA is isosceles. DF has to be congruent to EF because they are the same length. So, AF has to be the angle bisector.

Andre's rough-draft justification: What if you draw a segment from F to A? Segments DF and EF are congruent. Also, angle DAF is congruent to angle EAF. Then both triangles are congruent on either side of the angle bisector line.

1. Each student tried to justify why their construction worked. With your partner, discuss each student's approach.
    - What do you notice that this student understands about the problem?
    - What question would you ask them to help them move forward?
2. Using the ideas you heard and the ways that each student could make their explanation better, write your own explanation for why ray AF must be an angle bisector.

14.3: Reflecting on Reflection

1. Here is a diagram of an isosceles triangle APB with segment AP congruent to segment BP.
   Here is a valid proof that the angle bisector of the vertex angle of an isosceles triangle is a line of symmetry.

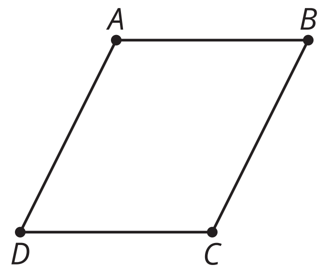


   a. Read the proof and annotate the diagram with each piece of information in the proof.

   b. Write a summary of how this proof shows the angle bisector of the vertex angle of an isosceles triangle is a line of symmetry.

   - Segment AP is congruent to segment BP because triangle APB is isosceles.
   - The angle bisector of APB intersects segment AB. Call that point Q.
   - By the definition of angle bisector, angles APQ and BPQ are congruent.
   - Segment PQ is congruent to itself.
   - By the Side-Angle-Side Triangle Congruence Theorem, triangle APQ must be congruent to triangle BPQ.
   - Therefore the corresponding segments AQ and BQ are congruent and corresponding angles AQP and BQP are congruent.
   - Since angles AQP and BQP are both congruent and supplementary angles, each angle must be a right angle.
   - So PQ must be the perpendicular bisector of segment AB.
   - Because reflection across perpendicular bisectors takes segments onto themselves and swaps the endpoints, when we reflect the triangle across PQ the vertex P will stay in the same spot and the 2 endpoints of the base, A and B, will switch places.
   - Therefore the angle bisector PQ is a line of symmetry for triangle APB.

2. Here is a diagram of parallelogram $ABCD$.
Here is an invalid proof that a diagonal of a parallelogram is a line of symmetry.



a. Read the proof and annotate the diagram with each piece of information in the proof.

b. Find the errors that make this proof invalid. Highlight any lines that have errors or false assumptions.

- The diagonals of a parallelogram intersect. Call that point M.
- The diagonals of a parallelogram bisect each other, so MB is congruent to MD.
- By the definition of parallelogram, the opposite sides AB and CD are parallel.
- Angles ABM and ADM are alternate interior angles of parallel lines so they must be congruent.
- Segment AM is congruent to itself.
- By the Side-Angle-Side Triangle Congruence Theorem, triangle ABM is congruent to triangle ADM.
- Therefore the corresponding angles AMB and AMD are congruent.
- Since angles AMB and AMD are both congruent and supplementary angles, each angle must be a right angle.
- So AC must be the perpendicular bisector of segment BD.
- Because reflection across perpendicular bisectors takes segments onto themselves and swaps the endpoints, when we reflect the parallelogram across AC the vertices A and C will stay in the same spot and the 2 endpoints of the other diagonal, B and D, will switch places.
- Therefore diagonal AC is a line of symmetry for parallelogram $ABCD$.

Are you ready for more?

There are quadrilaterals for which the diagonals are lines of symmetry.

1. What is an example of such a quadrilateral?

2. How would you modify this proof to be a valid proof for that type of quadrilateral?

Lesson 14 Summary

Earlier we constructed an angle bisector, but we did not prove that the construction always works. Now that we know more we can see why each step is necessary for the construction to precisely bisect an angle. The proof uses some ideas from constructions:

- The midpoint of a segment divides the segment into 2 congruent segments.
- All the radii of a given circle are congruent.

But it also uses some ideas from triangle congruence:

- If triangles have 2 pairs of sides and the angle between them congruent, then the triangles are congruent.
- If triangles are congruent, then the corresponding parts of those triangles are also congruent.

Triangle congruence theorems and properties of rigid transformations can be useful for proving many things, including constructions.