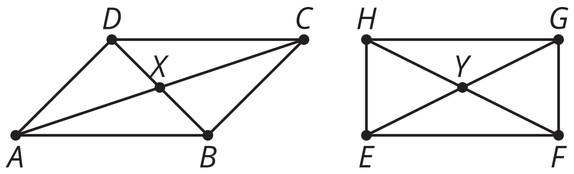
Lesson 13: Proofs about Parallelograms

- Let’s prove theorems about parallelograms.

13.1: Notice and Wonder: Diagonals

Here is parallelogram $ABCD$ and rectangle $EFGH$. What do you notice? What do you wonder?



13.2: The Diagonals of a Parallelogram

Conjecture: The diagonals of a parallelogram bisect each other.

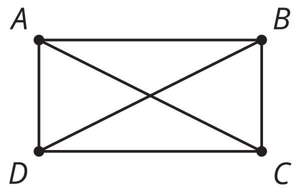
1. Use the tools available to convince yourself the conjecture is true.

2. Convince your partner that the conjecture is true for any parallelogram. Can the 2 of you think of different ways to convince each other?

3. What information is needed to prove that the diagonals of a parallelogram bisect each other?

4. Prove that segment AC bisects segment BD, and that segment BD bisects segment AC.

13.3: Work Backwards to Prove



Given: $ABCD$ is a parallelogram with AB parallel to CD and AD parallel to BC. Diagonal AC is congruent to diagonal BD.

Prove: $ABCD$ is a rectangle (angles A, B, C, and D are right angles).

With your partner, you will work backwards from the statement to the proof until you feel confident that you can prove that $ABCD$ is a rectangle using only the given information.

Start with this sentence: I would know $ABCD$ is a rectangle if I knew ________________.
Then take turns saying this sentence: I would know [what my partner just said] if I knew ________________.

Write down what you each say. If you get to a statement and get stuck, go back to an earlier statement and try to take a different path.

Are you ready for more?

Two intersecting segments always make a quadrilateral if you connect the endpoints. What has to be true about the intersecting segments in order to make a(n):

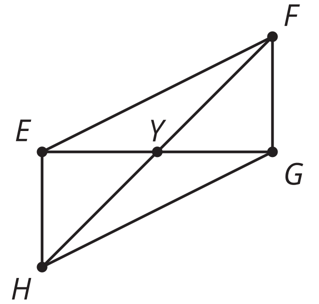
1. rectangle
2. rhombus
3. square
4. kite
5. isosceles trapezoid

Lesson 13 Summary

A quadrilateral is a parallelogram if and only if its diagonals bisect each other. The "if and only if" language means that both the statement and its *converse* are true. So we need to prove:

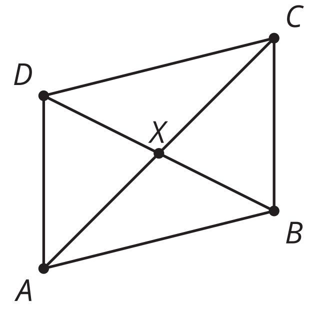
1. If a quadrilateral has diagonals that bisect each other, then it is a parallelogram.
2. If a quadrilateral is a parallelogram, then its diagonals bisect each other.

To prove part 1, make the statement specific: If quadrilateral $EFGH$ with diagonals EG and FH intersecting at Y so that EY is congruent to YG and FY is congruent to YH, then side EF is parallel to side GH and side EH is parallel to side FG.



We could prove triangles EYH and GYF are congruent by the Side-Angle-Side Triangle Congruence Theorem. That means that corresponding angles in the triangles are congruent, so angle YEH is congruent to YGF. This means that alternate interior angles formed by lines EH and FG are congruent, so lines EH and FG are parallel. We could also make an argument that shows triangles EYF and GYH are congruent, so that angles FEY and HGY are congruent, which means that lines EF and GH must be parallel.

To prove part 2, make the statement specific: If parallelogram $ABCD$ has side AB parallel to side CD and side AD parallel to side BC, and diagonals AC and BD that intersect at X, then we are trying to prove that X is the midpoint of AC and of BD.



We could use a transformation proof. Rotate parallelogram $ABCD$ by 180° using the midpoint of diagonal AC as the center of the rotation. Then show that the midpoint of diagonal AC is also the midpoint of diagonal BD. That point must be X since it is the only point on both line AC and line BD. So X must be the midpoints of both diagonals, meaning the diagonals bisect each other.

We have proved that any quadrilateral with diagonals that bisect each other is a parallelogram, and that any parallelogram has diagonals that bisect each other. Therefore, a quadrilateral is a parallelogram *if and only if* its diagonals bisect each other.