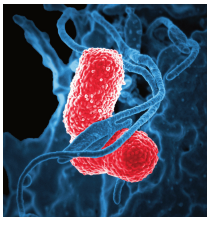
Lesson 1: Growing and Growing

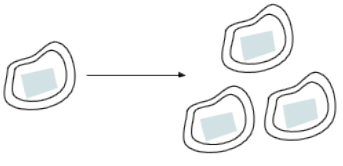
Let's choose the better deal.

1.1: Splitting Bacteria



There are some bacteria in a dish. Every hour, each bacterium splits into 3 bacteria.

1. This diagram shows a bacterium in hour 0 and then hour 1. Draw what happens in hours 2 and 3.



2. How many bacteria are there in hours 2 and 3?

1.2: A Genie in a Bottle

You are walking along a beach and your toe hits something hard. You reach down, grab onto a handle, and pull out a bottle! It is sandy. You start to brush it off with your towel. Poof! A genie appears.

He tells you, "Thank you for freeing me from that bottle! I was getting claustrophobic. You can choose one of these purses as a reward."

- Purse A which contains $1,000 today. If you leave it alone, it will contain $1,200 tomorrow (by magic). The next day, it will have $1,400. This pattern of $200 additional dollars per day will continue.
- Purse B which contains 1 penny today. Leave that penny in there, because tomorrow it will (magically) turn into 2 pennies. The next day, there will be 4 pennies. The amount in the purse will continue to double each day.

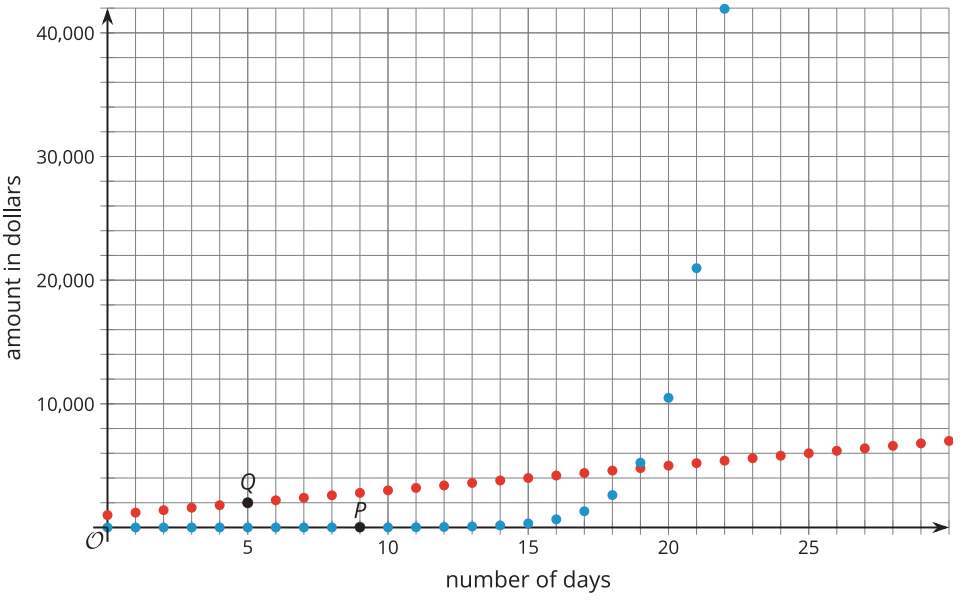
1. How much money will be in each purse after a week? After two weeks?

2. The genie later added that he will let the money in each purse grow for three weeks. How much money will be in each purse then?

3. Which purse contains more money after 30 days?

1.3: Graphing the Genie's Offer

Here are graphs showing how the amount of money in the purses changes. Remember Purse A starts with $1,000 and grows by $200 each day. Purse B starts with $0.01 and doubles each day.



1. Which graph shows the amount of money in Purse A? Which graph shows the amount of money in Purse B? Explain how you know.

2. Points P and Q are labeled on the graph. Explain what they mean in terms of the genie's offer.

3. What are the coordinates of the vertical intercept for each graph? Explain how you know.

4. When does Purse B become a better choice than Purse A? Explain your reasoning.

5. Knowing what you know now, which purse would you choose? Explain your reasoning.

Are you ready for more?

Okay, okay, the genie smiles, disappointed. I will give you an even *more* enticing deal. He explains that Purse B stays the same, but Purse A now increases by $250,000 every day. Which purse should you choose?

Lesson 1 Summary

When we repeatedly double a positive number, it eventually becomes *very* large. Let's start with 0.001. The table shows what happens when we begin to double:

0.001	0.002	0.004	0.008	0.016

If we want to continue this process, it is convenient to use an exponent. For example, the last entry in the table, 0.016, is 0.001 being doubled 4 times, or $(0.001) \cdot 2 \cdot 2 \cdot 2 \cdot 2$, which can be expressed as $(0.001) \cdot 2^4$.

Even though we started with a very small number, 0.001, we don't have to double it that many times to reach a very large number. For example, if we double it 30 times, represented by $(0.001) \cdot 2^{30}$, the result is greater than 1,000,000.

Throughout this unit, we will look at many situations where quantities grow or decrease by applying the same factor repeatedly.