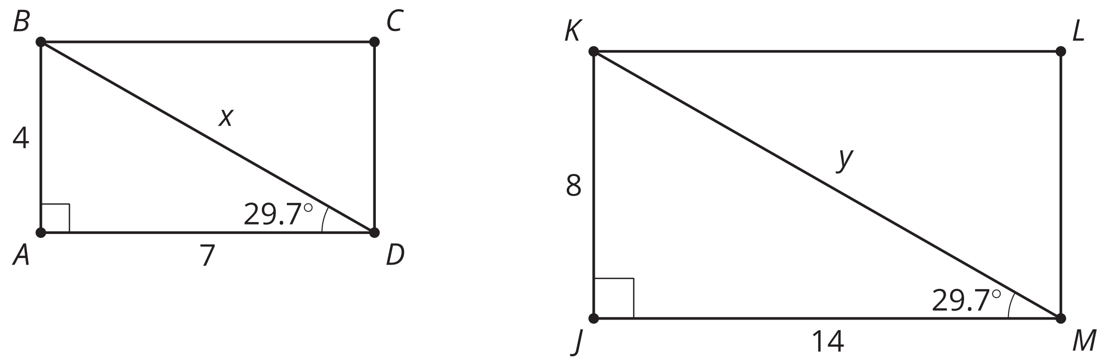
Lesson 2: Half a Square

- Let’s investigate the properties of diagonals of squares.

2.1: Diagonals of Rectangles



Calculate the values of x and y.

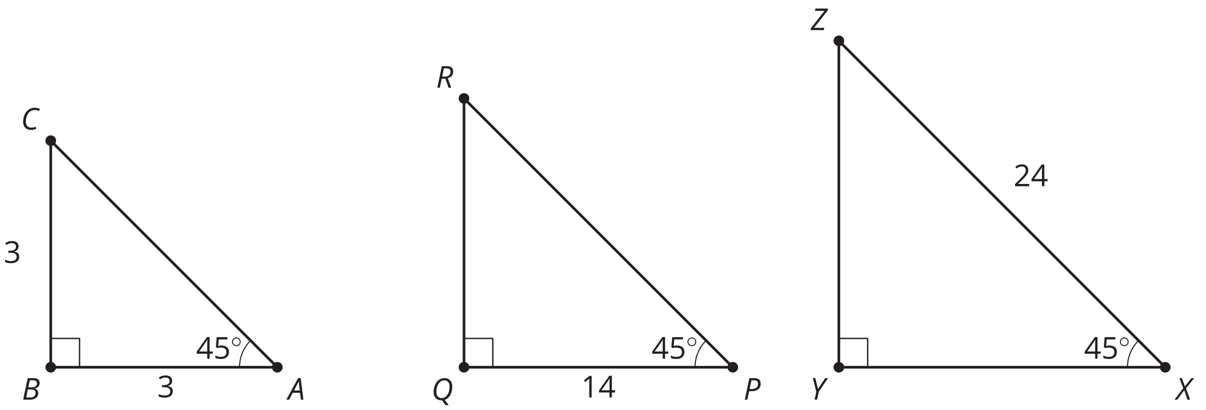
2.2: Decomposing Squares

1. Draw a square with side lengths of 1 cm. Estimate the length of the diagonal. Then calculate the length of the diagonal.

2. Measure the side length and diagonal length of several squares, in centimeters. Compute the ratio of side to diagonal length for each.

3. Make a conjecture.

2.3: Generalize Half Squares



Calculate the lengths of the 5 unlabeled sides.

Are you ready for more?

Square $ABCD$ has a diagonal length of x and side length of s. Rhombus $EFGH$ has side length s.

1. How do the diagonals of $EFGH$ compare to the diagonals of $ABCD$?

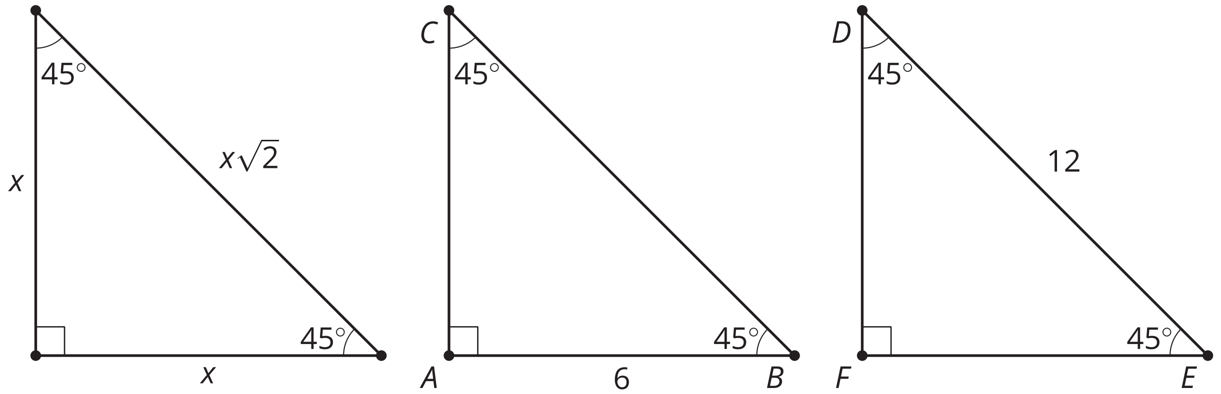
2. What is the maximum possible length of a diagonal of a rhombus of side length s?

Lesson 2 Summary

Drawing the diagonal of a square decomposes the square into 2 congruent triangles. They are right isosceles triangles with acute angles of 45 degrees. These congruent angles make all right isosceles triangles similar by the Angle-Angle Triangle Similarity Theorem.

Consider an isosceles right triangle with legs 1 unit long where c is the length of the hypotenuse. By the Pythagorean Theorem, we can say $1^2 + 1^2 = c^2$ so $c = \sqrt{2}$. The hypotenuse of an isosceles right triangle with legs 1 unit long is $\sqrt{2}$ units long.

Now, consider an isosceles right triangle with legs x units long. By the Angle-Angle Triangle Similarity Theorem, the triangle is similar to the isosceles right triangle with side lengths of 1, 1, and $\sqrt{2}$ units. A scale factor of x takes the triangle with leg length of 1 to the triangle with leg length of x. Therefore, the hypotenuse of the isosceles right triangle with legs x units long is $x\sqrt{2}$ units long.



In triangle ABC, $x = 6$ so AC is 6 units long and BC is $6\sqrt{2}$ units long.

In triangle DEF, $x\sqrt{2} = 12$ so $x = \frac{12}{\sqrt{2}}$, which means both EF and DF are $\frac{12}{\sqrt{2}}$ units long.