Lesson 14: Using Diagrams to Represent Addition and Subtraction

Let's represent addition and subtraction of decimals.

14.1: Do the Zeros Matter?

1. Evaluate mentally: $1.009 + 0.391$

2. Decide if each equation is true or false. Be prepared to explain your reasoning.

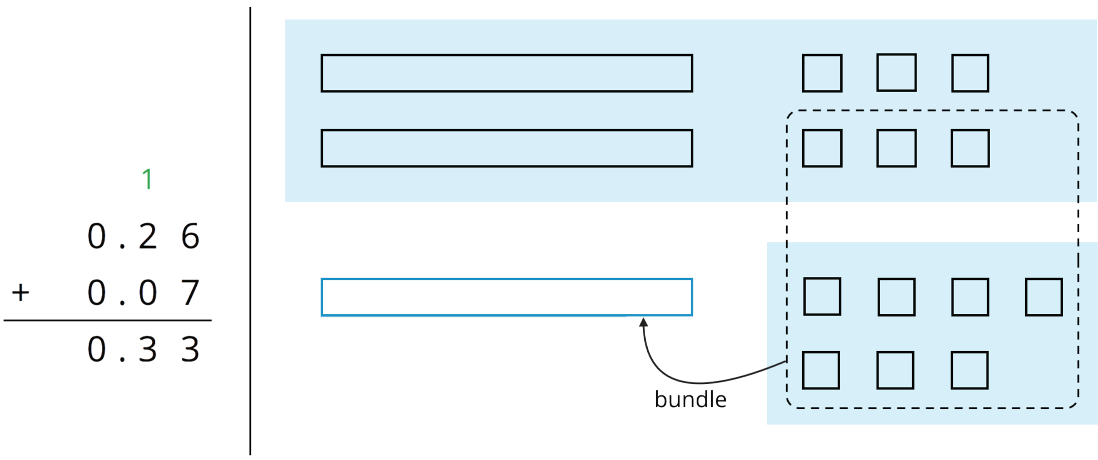
   a. $34.56000 = 34.56$

   b. $25 = 25.0$

   c. $2.405 = 2.45$

14.2: Finding Sums in Different Ways

1. Here are two ways to calculate the value of $0.26 + 0.07$. In the diagram, each rectangle represents 0.1 and each square represents 0.01.



   Use what you know about base-ten units and addition to explain:

   a. Why ten squares can be "bundled" into a rectangle.

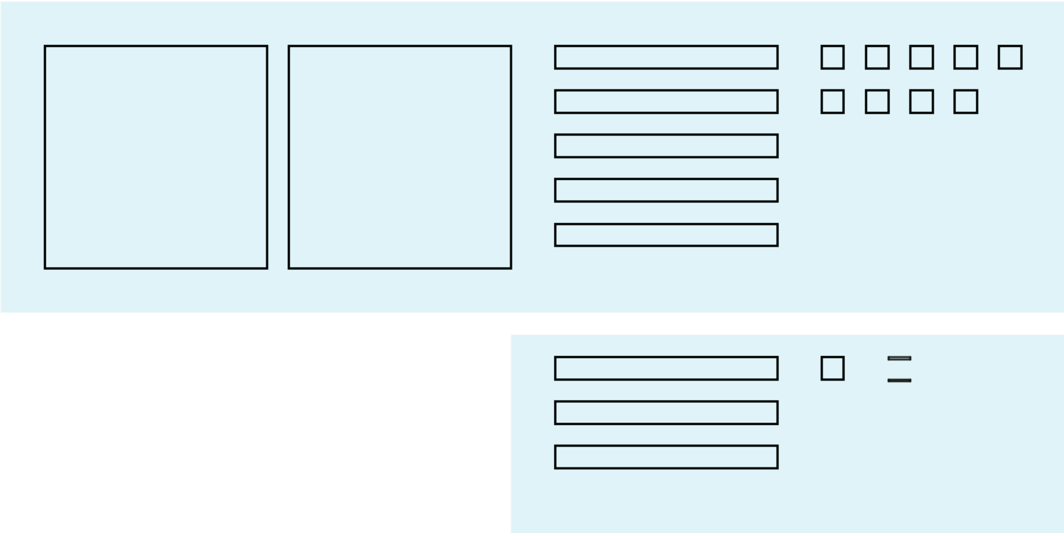
b. How this “bundling” is represented in the vertical calculation.

2. Find the value of $0.38 + 0.69$ by drawing a diagram. Can you find the sum without bundling? Would it be useful to bundle some pieces? Explain your reasoning.

3. Calculate $0.38 + 0.69$. Check your calculation against your diagram in the previous question.

4. Find each sum. The larger square represents 1.

   a.



   b.

$$\begin{array}{r} 6.03 \\ +\ 0.098 \\ \hline \end{array}$$

Are you ready for more?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

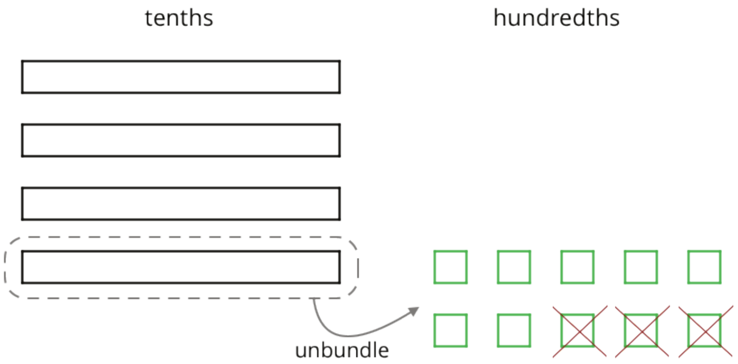
1. If you had 500 violet jewels and wanted to trade so that you carried as few jewels as possible, which jewels would you have?

2. Suppose you have 1 orange jewel, 2 yellow jewels, and 1 indigo jewel. If you're given 2 green jewels and 1 yellow jewels, what is the fewest number of jewels that could represent the value of the jewels you have?

14.3: Subtracting Decimals of Different Lengths

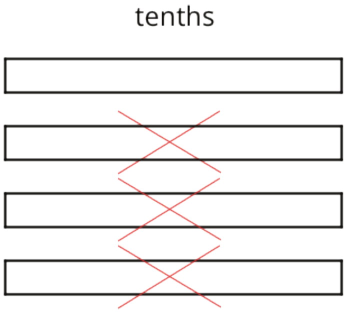
Diego and Noah drew different diagrams to represent $0.4 - 0.03$. Each rectangle represents 0.1. Each square represents 0.01.

- Diego started by drawing 4 rectangles to represent 0.4. He then replaced 1 rectangle with 10 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 3 rectangles and 7 squares in his diagram.



Diego's Method

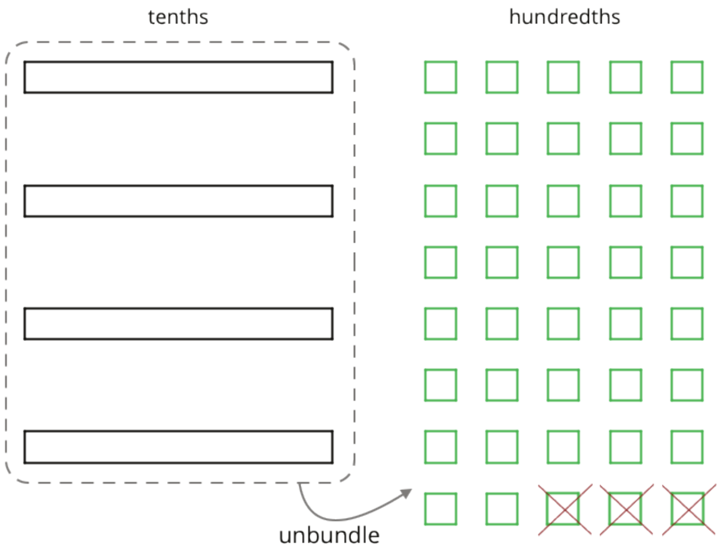
- Noah started by drawing 4 rectangles to represent 0.4. He then crossed out 3 rectangles to represent the subtraction, leaving 1 rectangle in his diagram.



Noah's Method

1. Do you agree that either diagram correctly represents $0.4 - 0.03$? Discuss your reasoning with a partner.

2. Elena also drew a diagram to represent $0.4 - 0.03$. She started by drawing 4 rectangles. She then replaced all 4 rectangles with 40 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 37 squares in her diagram. Is her diagram correct? Discuss your reasoning with a partner.



Elena's Method

3. Find each difference. Explain or show your reasoning.

   a. $0.3 - 0.05$

   b. $2.1 - 0.4$

c. $1.03 - 0.06$

d. $0.02 - 0.007$

Are you ready for more?

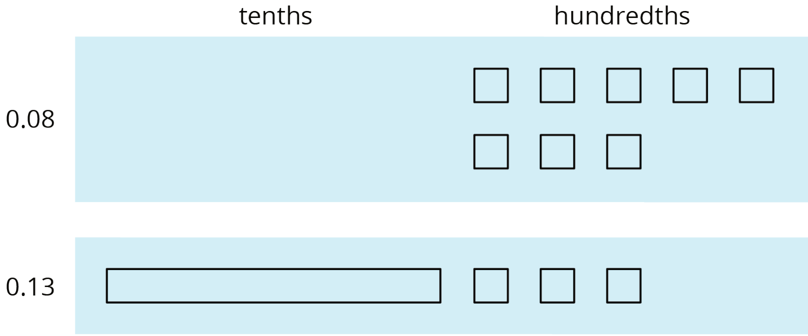
A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

At the Auld Shoppe, a shopper buys items that are worth 2 yellow jewels, 2 green jewels, 2 blue jewels, and 1 indigo jewel. If they came into the store with 1 red jewel, 1 yellow jewel, 2 green jewels, 1 blue jewel, and 2 violet jewels, what jewels do they leave with? Assume the shopkeeper gives them their change using as few jewels as possible.

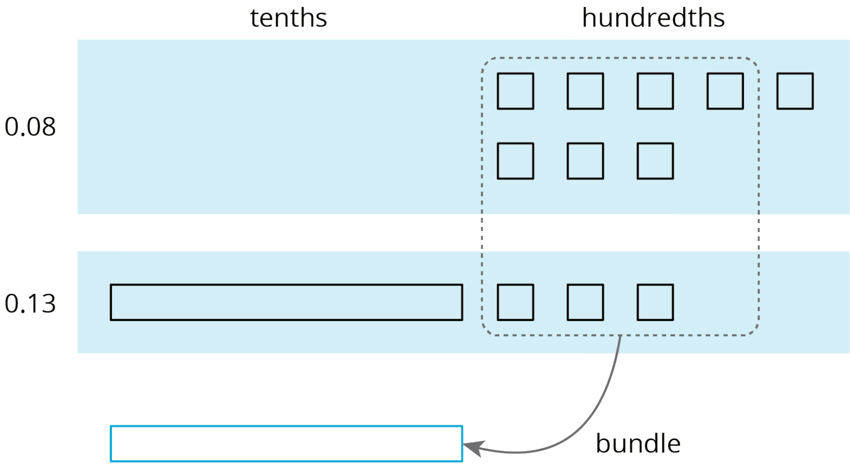
Lesson 14 Summary

Base-ten diagrams represent collections of base-ten units—tens, ones, tenths, hundredths, etc. We can use them to help us understand sums of decimals.

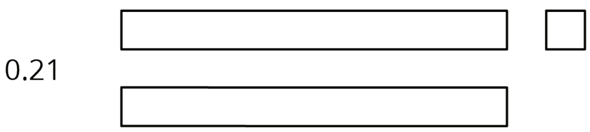
Suppose we are finding $0.08 + 0.13$. Here is a diagram where a square represents 0.01 and a rectangle (made up of ten squares) represents 0.1.



To find the sum, we can "bundle" (or compose) 10 hundredths as 1 tenth.



We now have 2 tenths and 1 hundredth, so $0.08 + 0.13 = 0.21$.

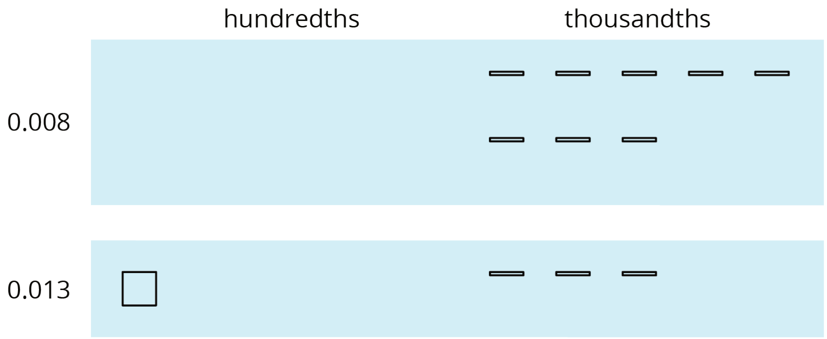


We can also use vertical calculation to find $0.08 + 0.13$.

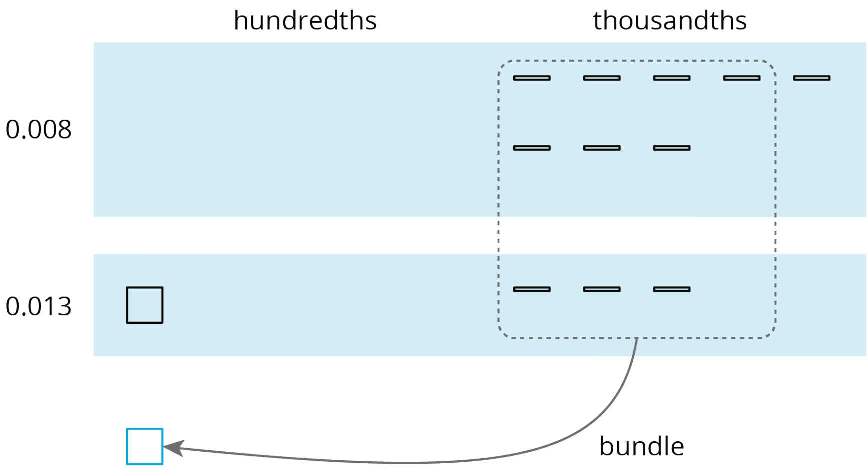
$$\begin{array}{r} {\scriptstyle 1} \\ 0.13 \\ +\ 0.08 \\ \hline 0.21 \end{array}$$

Notice how this representation also shows 10 hundredths are bundled (or composed) as 1 tenth.

This works for any decimal place. Suppose we are finding $0.008 + 0.013$. Here is a diagram where a small rectangle represents 0.001.



We can “bundle” (or compose) 10 thousandths as 1 hundredth.

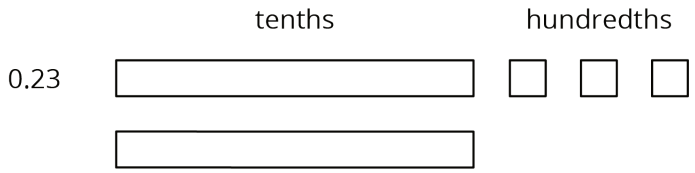


The sum is 2 hundredths and 1 thousandth.

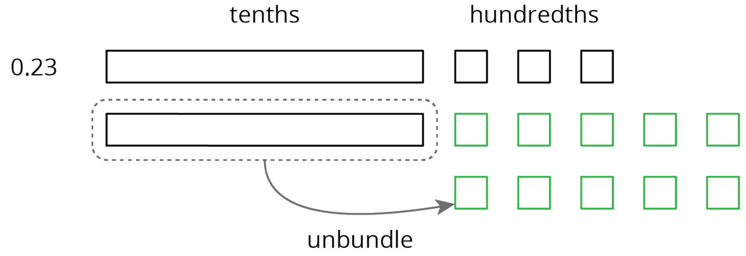
Here is a vertical calculation of $0.008 + 0.013$.

$$\begin{array}{r} {}^{1} \\ 0.013 \\ +\ 0.008 \\ \hline 0.021 \end{array}$$

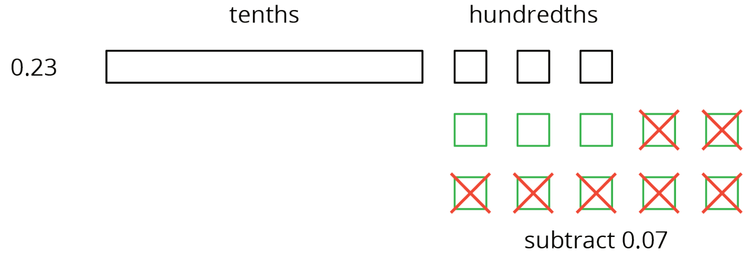
Base-ten diagrams can help us understand subtraction as well. Suppose we are finding $0.23 - 0.07$. Here is a diagram showing 0.23, or 2 tenths and 3 hundredths.



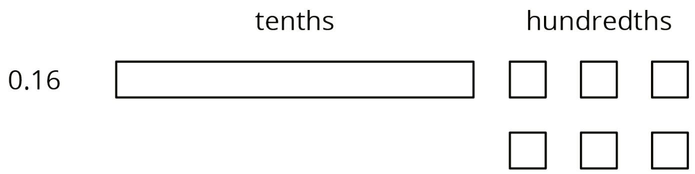
Subtracting 7 hundredths means removing 7 small squares, but we do not have enough to remove. Because 1 tenth is equal to 10 hundredths, we can “unbundle” (or decompose) one of the tenths (1 rectangle) into 10 hundredths (10 small squares).



We now have 1 tenth and 13 hundredths, from which we can remove 7 hundredths.



We have 1 tenth and 6 hundredths remaining, so $0.23 - 0.07 = 0.16$.

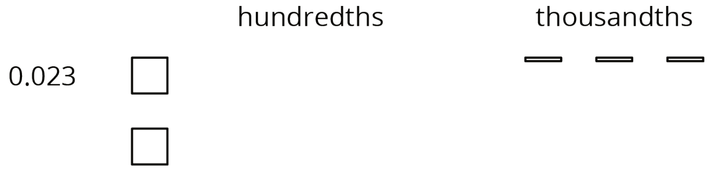


Here is a vertical calculation of $0.23 - 0.07$.

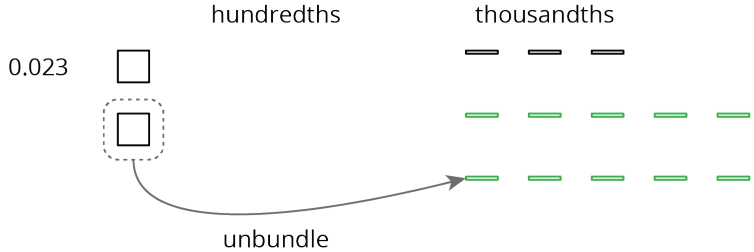
$$\begin{array}{rcccc} & & & 1 & 13 \\ & 0 & . & \cancel{2} & \cancel{3} \\ - & 0 & . & 0 & 7 \\ \hline & 0 & . & 1 & 6 \end{array}$$

Notice how this representation also shows a tenth is unbundled (or decomposed) into 10 hundredths in order to subtract 7 hundredths.

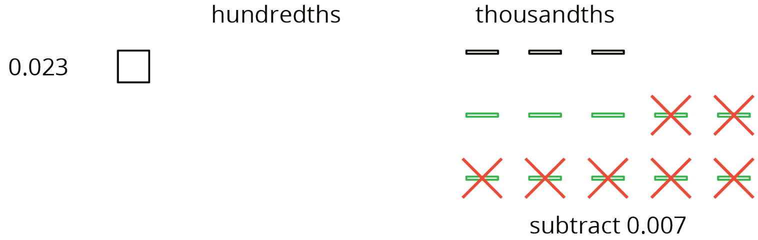
This works for any decimal place. Suppose we are finding $0.023 - 0.007$. Here is a diagram showing 0.023.



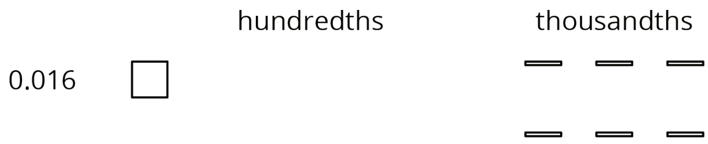
We want to remove 7 thousandths (7 small rectangles). We can "unbundle" (or decompose) one of the hundredths into 10 thousandths.



Now we can remove 7 thousandths.



We have 1 hundredth and 6 thousandths remaining, so $0.023 - 0.007 = 0.016$.



Here is a vertical calculation of $0.023 - 0.007$.

$$
\begin{array}{r}
 \,1\ \ 13 \\
 0.0\ \not{2}\ \ \not{3} \\
 -\quad 0.0\ \ 0\ \ 7 \\
 \hline
 0.0\ \ 1\ \ 6
\end{array}
$$