## Lesson 13: Completing the Square (Part 2)

* Let’s solve some harder quadratic equations.

### 13.1: Math Talk: Equations with Fractions

Solve each equation mentally.

$x+x=\frac{1}{4}$

$\left(\frac{3}{2}\right)^{2}=x$

$\frac{3}{5}+x=\frac{9}{5}$

$\frac{1}{12}+x=\frac{1}{4}$

### 13.2: Solving Some Harder Equations

Solve these equations by completing the square.

1. $\left(x−3\right)\left(x+1\right)=5$
2. $x^{2}+\frac{1}{2}x=\frac{3}{16}$
3. $x^{2}+3x+\frac{8}{4}=0$
4. $\left(7−x\right)\left(3−x\right)+3=0$
5. $x^{2}+1.6x+0.63=0$

#### Are you ready for more?

1. Show that the equation $x^{2}+10x+9=0$ is equivalent to $\left(x+3\right)^{2}+4x=0$.
2. Write an equation that is equivalent to $x^{2}+9x+16=0$ and that includes $\left(x+4\right)^{2}$.
3. Does this method help you find solutions to the equations? Explain your reasoning.

### 13.3: Spot Those Errors!

Here are four equations, followed by worked solutions of the equations. Each solution has at least one error.

* Solve one or more of these equations by completing the square.
* Then, look at the worked solution of the same equation as the one you solved. Find and describe the error or errors in the worked solution.
1. $x^{2}+14x=-24$
2. $x^{2}−10x+16=0$
3. $x^{2}+2.4x=-0.8$
4. $x^{2}−\frac{6}{5}x+\frac{1}{5}=0$

Worked solutions (with errors):

1.

$\begin{matrix}x^{2}+14x&=-24\\x^{2}+14x+28&=4\\\left(x+7\right)^{2}&=4\\&\\x+7=2 &or x+7=-2\\x=-5 &or x=-9\end{matrix}$

2.

$\begin{matrix}x^{2}−10x+16&=0\\x^{2}−10x+25&=9\\\left(x−5\right)^{2}&=9\\&\\x−5=9 &or x−5=-9\\x=14 &or x=-4\end{matrix}$

3.

$\begin{matrix}x^{2}+2.4x&=-0.8\\x^{2}+2.4x+1.44&=0.64\\\left(x+1.2\right)^{2}&=0.64\\x+1.2&=0.8\\x&=-0.4\end{matrix}$

4.

$\begin{matrix}x^{2}−\frac{6}{5}x+\frac{1}{5}&=0\\x^{2}−\frac{6}{5}x+\frac{9}{25}&=\frac{9}{25}\\\left(x−\frac{3}{5}\right)^{2}&=\frac{9}{25}\\&\\x−\frac{3}{5}=\frac{3}{5} &or x−\frac{3}{5}=-\frac{3}{5}\\x=\frac{6}{5} &or x=0\end{matrix}$

### Lesson 13 Summary

Completing the square can be a useful method for solving quadratic equations in cases in which it is not easy to rewrite an expression in factored form. For example, let’s solve this equation:

$x^{2}+5x−\frac{75}{4}=0$

First, we’ll add $\frac{75}{4}$ to each side to make things easier on ourselves.

$\begin{matrix}x^{2}+5x−\frac{75}{4}+\frac{75}{4}&=0+\frac{75}{4}\\x^{2}+5x&=\frac{75}{4}\end{matrix}$

To complete the square, take $\frac{1}{2}$ of the coefficient of the linear term 5, which is $\frac{5}{2}$, and square it, which is $\frac{25}{4}$. Add this to each side:

$\begin{matrix}x^{2}+5x+\frac{25}{4}&=\frac{75}{4}+\frac{25}{4}\\x^{2}+5x+\frac{25}{4}&=\frac{100}{4}\end{matrix}$

Notice that $\frac{100}{4}$ is equal to 25 and rewrite it:

$x^{2}+5x+\frac{25}{4}=25$

Since the left side is now a perfect square, let’s rewrite it:

$\left(x+\frac{5}{2}\right)^{2}=25$

For this equation to be true, one of these equations must true:

$x+\frac{5}{2}=5 or x+\frac{5}{2}=-5$

To finish up, we can subtract $\frac{5}{2}$ from each side of the equal sign in each equation.

$\begin{matrix}x=5−\frac{5}{2} &or x=-5−\frac{5}{2}\\x=\frac{5}{2} &or x=-\frac{15}{2}\\x=2\frac{1}{2} &or x=-7\frac{1}{2}\end{matrix}$

It takes some practice to become proficient at completing the square, but it makes it possible to solve many more equations than you could by methods you learned previously.



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