Lesson 10: Connecting Equations to Graphs (Part 1)

- Let's investigate what graphs can tell us about the equations and relationships they represent.

10.1: Games and Rides

Jada has $20 to spend on games and rides at a carnival. Games cost $1 each and rides are $2 each.

1. Which equation represents the relationship between the number of games, x, and the number of rides, y, that Jada could do if she spends all her money?

   A: $x + y = 20$ B: $2x + y = 20$ C: $x + 2y = 20$

2. Explain what each of the other two equations could mean in this situation.

10.2: Graphing Games and Rides

Here are the three equations. Each represents the relationship between the number of games, x, the number of rides, y, and the dollar amount a student is spending on games and rides at a different amusement park.

Equation 1: $x + y = 20$

Equation 2: $2.50x + y = 15$

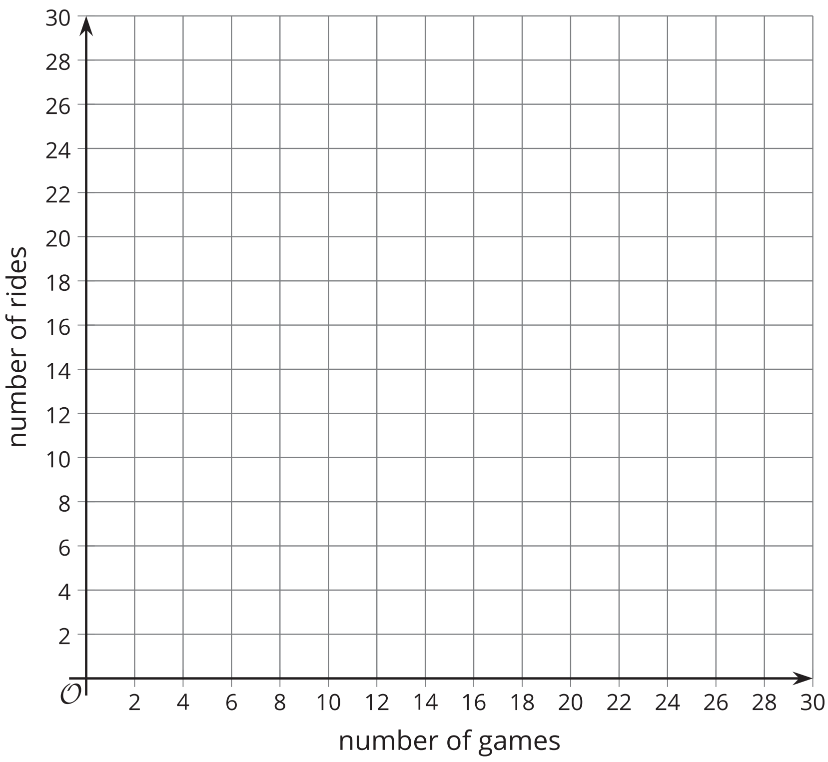
Equation 3: $x + 4y = 28$

Your teacher will assign to you (or ask you to choose) 1–2 equations. For each assigned (or chosen) equation, answer the questions.

First equation: ______________________________

1. What's the number of rides the student could get on if they don't play any games? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.

2. What's the number of games the student could play if they don't get on any rides? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.



3. Draw a line to connect the two points you've drawn.

4. Complete the sentences: "If the student played no games, they can get on __________ rides. For every additional game that the student plays, x, the possible number of rides, y, ____________________ (increases or decreases) by __________."

5. What is the slope of your graph? Where does the graph intersect the vertical axis?

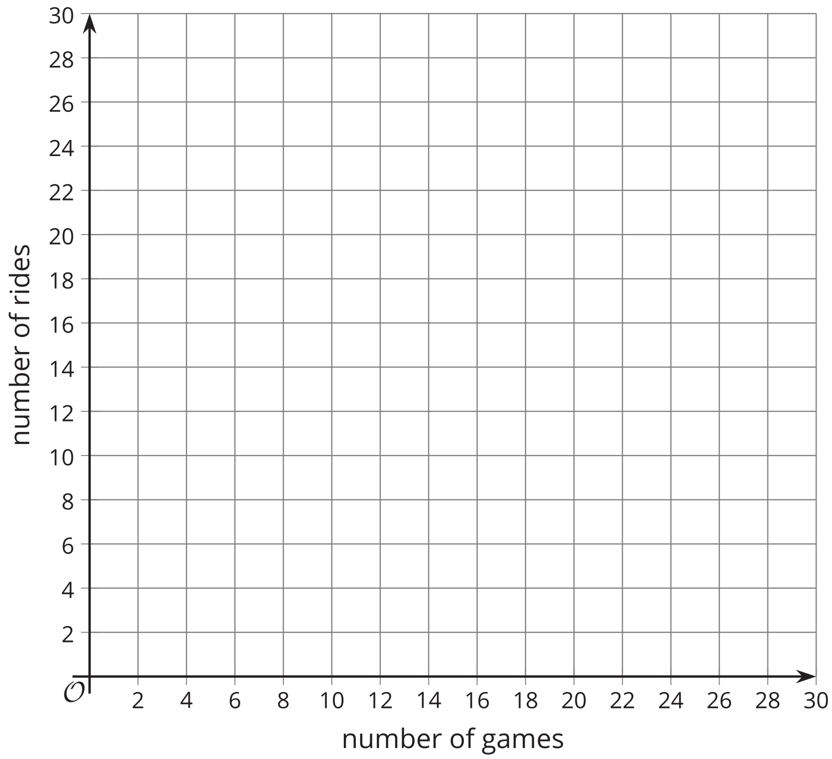
6. Rearrange the equation to solve for y.

7. What connections, if any, do you notice between your new equation and the graph?

Second equation: ______________________

1. What's the number of rides the student could get on if they don't play any games? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.

2. What's the number of games the student could play if they don't get on any rides? On the coordinate plane, mark the point that represents this situation and label the point with its coordinates.



3. Draw a line to connect the two points you've drawn.

4. Complete the sentences: "If the student played no games, they can get on __________ rides. For every additional game that a student plays, x, the possible number of rides, y, ____________________ (increases or decreases) by __________."

5. What is the slope of your graph? Where does the graph intersect the vertical axis?

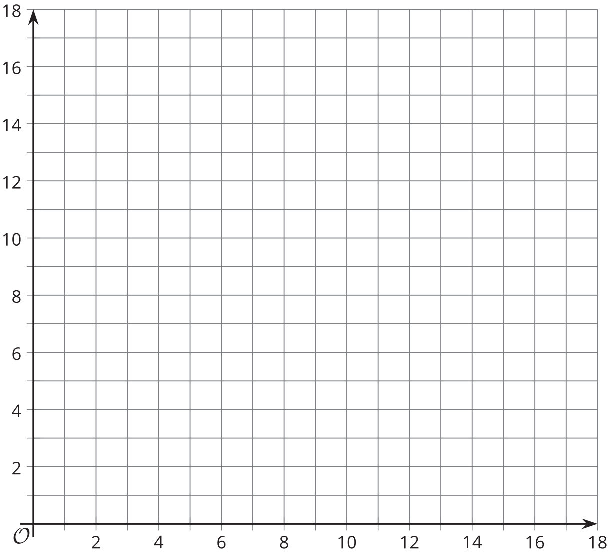
6. Rearrange the equation to solve for y.

7. What connections, if any, do you notice between your new equation and the graph?

10.3: Nickels and Dimes

Andre's coin jar contains 85 cents. There are no quarters or pennies in the jar, so the jar has all nickels, all dimes, or some of each.

1. Write an equation that relates the number of nickels, n, the number of dimes, d, and the amount of money, in cents, in the coin jar.



2. Graph your equation on the coordinate plane. Be sure to label the axes.

3. How many nickels are in the jar if there are no dimes?

4. How many dimes are in the jar if there are no nickels?

Are you ready for more?

What are all the different ways the coin jar could have 85 cents if it could also contain quarters?

Lesson 10 Summary

Linear equations can be written in different forms. Some forms allow us to better see the relationship between quantities or to predict the graph of the equation.

Suppose an athlete wishes to burn 700 calories a day by running and swimming. He burns 17.5 calories per minute of running and 12.5 calories per minute of freestyle swimming.

Let x represents the number of minutes of running and y the number of minutes of swimming. To represent the combination of running and swimming that would allow him to burn 700 calories, we can write:

$$17.5x + 12.5y = 700$$

We can reason that the more minutes he runs, the fewer minutes he has to swim to meet his goal. In other words, as x increases, y decreases. If we graph the equation, the line will slant down from left to right.

If the athlete only runs and doesn't swim, how many minutes would he need to run?

Let's substitute 0 for y to find x:

$$\begin{aligned} 17.5x + 12.5(0) &= 700 \\ 17.5x &= 700 \\ x &= \frac{700}{17.5} \\ x &= 40 \end{aligned}$$

On a graph, this combination of times is the point $(40, 0)$, which is the x-intercept.

If he only swims and doesn't run, how many minutes would he need to swim?

Let's substitute 0 for x to find y:

$$\begin{aligned} 17.5(0) + 12.5y &= 700 \\ 12.5y &= 700 \\ y &= \frac{700}{12.5} \\ y &= 56 \end{aligned}$$

On a graph, this combination of times is the point $(0, 56)$, which is the y-intercept.

If the athlete wants to know how many minutes he would need to swim if he runs for 15 minutes, 20 minutes, or 30 minutes, he can substitute each of these values for x in the equation and find y. Or, he can first solve the equation for y:

$$\begin{aligned} 17.5x + 12.5y &= 700 \\ 12.5y &= 700 - 17.5x \\ y &= \frac{700 - 17.5x}{12.5} \\ y &= 56 - 1.4x \end{aligned}$$

Notice that $y = 56 - 1.4x$, or $y = \text{-}1.4x + 56$, is written in slope-intercept form.

- The coefficient of x, -1.4, is the slope of the graph. It means that as x increases by 1, y falls by 1.4. For every additional minute of running, the athlete can swim 1.4 fewer minutes.

- The constant term, 56, tells us where the graph intersects the y-axis. It tells us the number minutes the athlete would need to swim if he does no running.

The first equation we wrote, $17.5x + 12.5y = 700$, is a linear equation in standard form. In general, it is expressed as $Ax + By = C$, where x and y are variables, and A, B, and C are numbers.

The two equations, $17.5x + 12.5y = 700$ and $y = \text{-}1.4x + 56$, are equivalent, so they have the same solutions and the same graph.