## Lesson 16: The Quadratic Formula

* Let’s learn a formula for finding solutions to quadratic equations.

### 16.1: Evaluate It

Each expression represents two numbers. Evaluate the expressions and find the two numbers.

1. $1\pm \sqrt{49}$
2. $\frac{8\pm 2}{5}$
3. $\pm \sqrt{\left(-5\right)^{2}−4⋅4⋅1}$
4. $\frac{-18\pm \sqrt{36}}{2⋅3}$

### 16.2: Pesky Equations

Choose one equation to solve, either by rewriting it in factored form or by completing the square. Be prepared to explain your choice of method.

1. $x^{2}−2x−1.25=0$
2. $5x^{2}+9x−44=0$
3. $x^{2}+1.25x=0.375$
4. $4x^{2}−28x+29=0$

### 16.3: Meet the Quadratic Formula

Here is a formula called the **quadratic formula**.

$x=\frac{-b\pm \sqrt{b^{2}−4ac}}{2a}$

The formula can be used to find the solutions to any quadratic equation in the form of $ax^{2}+bx+c=0$, where $a$, $b$, and $c$ are numbers and $a$ is not 0.

This example shows how it is used to solve $x^{2}−8x+15=0$, in which $a=1$, $b=-8$, and $c=15$.

$\begin{matrix}x&=\frac{-b\pm \sqrt{b^{2}−4ac}}{2a}&  &original equation&\\x&=\frac{-\left(-8\right)\pm \sqrt{\left(-8\right)^{2}−4\left(1\right)\left(15\right)}}{2\left(1\right)}&  &substitute the values of a,b,and c&\\x&=\frac{8\pm \sqrt{64−60}}{2}&  &evaluate each part of the expression&\\x&=\frac{8\pm \sqrt{4}}{2}&&&\\x&=\frac{8\pm 2}{2}&&&\\x&=\frac{10}{2}  or  x=\frac{6}{2}&&&\\x&= 5   or  x= 3&&&\end{matrix}$

Here are some quadratic equations and their solutions. Use the quadratic formula to show that the solutions are correct.

1. $x^{2}+4x−5=0$. The solutions are $x=-5$ and $x=1$.
2. $x^{2}+7x+12=0$. The solutions are $x=-3$ and $x=-4$.
3. $x^{2}+10x+18=0$. The solutions are $x=-5\pm \frac{\sqrt{28}}{2}$.
4. $x^{2}−8x+11=0$. The solutions are $x=4\pm \frac{\sqrt{20}}{2}$.
5. $9x^{2}−6x+1=0$. The solution is $x=\frac{1}{3}$.
6. $6x^{2}+9x−15=0$. The solutions are $x=-\frac{5}{2}$ and $x=1$.

#### Are you ready for more?

1. Use the quadratic formula to solve $ax^{2}+c=0$. Let’s call the resulting equation P.
2. Solve the equation $3x^{2}−27=0$ in two ways, showing your reasoning for each:
	* Without using any formulas.
	* Using equation P.
3. Check that you got the same solutions using each method.
4. Use the quadratic formula to solve $ax^{2}+bx=0$. Let’s call the resulting equation Q.
5. Solve the equation $2x^{2}+5x=0$ in two ways, showing your reasoning for each:
	* Without using any formulas.
	* Using equation Q.
6. Check that you got the same solutions using each method.

### Lesson 16 Summary

We have learned a couple of methods for solving quadratic equations algebraically:

* by rewriting the equation as $factored form=0$ and using the zero product property
* by completing the square

Some equations can be solved quickly with one of these methods, but many cannot. Here is an example: $5x^{2}−3x−1=0$. The expression on the left cannot be rewritten in factored form with rational coefficients. Because the coefficient of the squared term is not a perfect square, and the coefficient of the linear term is an odd number, completing the square would be inconvenient and would result in a perfect square with fractions.

The **quadratic formula** can be used to find the solutions to any quadratic equation, including those that are tricky to solve with other methods.

For an equation of the form $ax^{2}+bx+c=0$, where $a$, $b$, and $c$ are numbers and $a\ne 0$, the solutions are given by:

$x=\frac{-b\pm \sqrt{b^{2}−4ac}}{2a}$

For the equation $5x^{2}−3x−1=0$, we see that $a=5$, $b=-3$, and $c=-1$. Let’s solve it!

$\begin{matrix}x&=\frac{-b\pm \sqrt{b^{2}−4ac}}{2a}&  &the quadratic formula&\\x&=\frac{-\left(-3\right)\pm \sqrt{\left(-3\right)^{2}−4\left(5\right)\left(-1\right)}}{2\left(5\right)}&  &substitute the values of a,b,and c&\\x&=\frac{3\pm \sqrt{9+20}}{10}&  &evaluate each part of the expression&\\x&=\frac{3\pm \sqrt{29}}{10}&&&\end{matrix}$

A calculator gives approximate solutions of 0.84 and -0.24 for $\frac{3+\sqrt{29}}{10}$ and $\frac{3−\sqrt{29}}{10}$.

We can also use the formula for simpler equations like $x^{2}−9x+8=0$, but it may not be the most efficient way. If the quadratic expression can be easily rewritten in factored form or made into a perfect square, those methods may be preferable. For example, rewriting $x^{2}−9x+8=0$ as $\left(x−1\right)\left(x−8\right)=0$ immediately tells us that the solutions are 1 and 8.



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