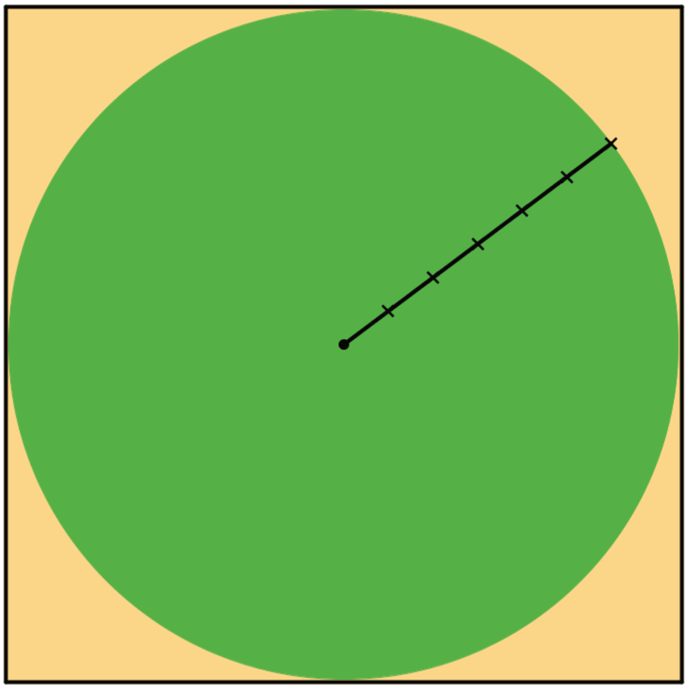
Lesson 15: Area of a Circle

Let's investigate the areas of circles.

15.1: Irrigating a Field

A circular field is set into a square with an 800 m side length. Estimate the field's area.



- About 5,000 m^2
- About 50,000 m^2
- About 500,000 m^2
- About 5,000,000 m^2
- About 50,000,000 m^2

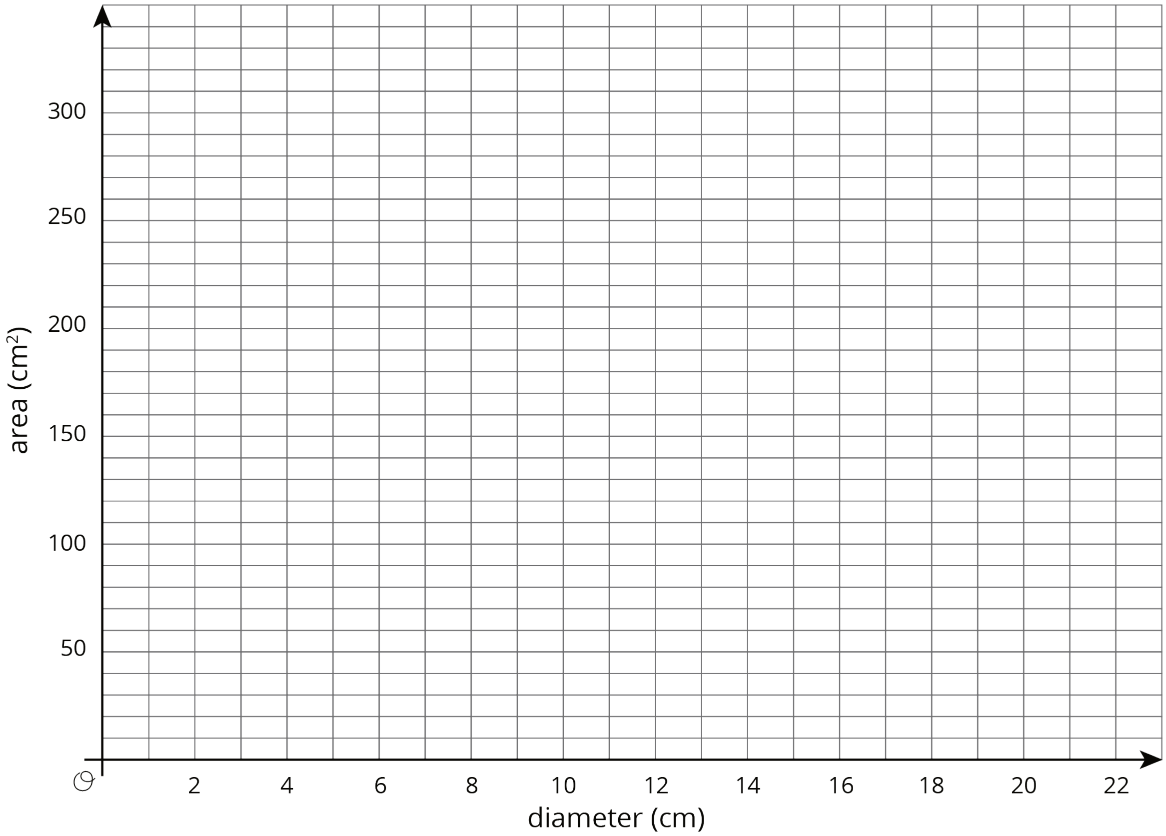
15.2: Estimating Areas of Circles

Your teacher will give your group two circles of different sizes.

1. For each circle, use the squares on the graph paper to measure the diameter and estimate the **area of the circle**. Record your measurements in the table.

diameter (cm)	estimated area (cm^2)

2. Plot the values from the table on the class coordinate plane. Then plot the class's data points on your coordinate plane.



3. In a previous lesson, you graphed the relationship between the diameter and circumference of a circle. How is this graph the same? How is it different?

Are you ready for more?

How many circles of radius 1 unit can you fit inside each of the following so that they do not overlap?

1. a circle of radius 2 units?

2. a circle of radius 3 units?

3. a circle of radius 4 units?

If you get stuck, consider using coins or other circular objects.

15.3: Making a Polygon out of a Circle

Your teacher will give you a circular object, a marker, and two pieces of paper of different colors.

Follow these instructions to create a visual display:

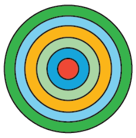
1. Using a thick marker, trace your circle in two separate places on the same piece of paper.
2. Cut out both circles, cutting around the marker line.
3. Fold and cut one of the circles into fourths.
4. Arrange the fourths so that straight sides are next to each other, but the curved edges are alternately on top and on bottom. Pause here so your teacher can review your work.
5. Fold and cut the fourths in half to make eighths. Arrange the eighths next to each other, like you did with the fourths.
6. If your pieces are still large enough, repeat the previous step to make sixteenths.
7. Glue the remaining circle and the new shape onto a piece of paper that is a different color.

After you finish gluing your shapes, answer the following questions.

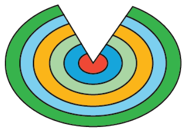
1. How do the areas of the two shapes compare?
2. What polygon does the shape made of the circle pieces most resemble?
3. How could you find the area of this polygon?

15.4: Making Another Polygon out of a Circle

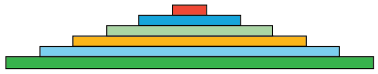
Imagine a circle made of rings that can bend, but not stretch.



A circle is made of rings.



The rings are unrolled.



The circle has been made into a new shape.

1. What polygon does the new shape resemble?

2. How does the area of the polygon compare to the area of the circle?

3. How can you find the area of the polygon?

4. Show, in detailed steps, how you could find the polygon’s area in terms of the circle’s measurements. Show your thinking. Organize it so it can be followed by others.

5. After you finish, trade papers with a partner and check each other’s work. If you disagree, work to reach an agreement. Discuss:
    - Do you agree or disagree with each step?
    - Is there a way to make the explanation clearer?

6. Return your partner’s work, and revise your explanation based on the feedback you received.

Lesson 15 Summary

The circumference C of a circle is proportional to the diameter d, and we can write this relationship as $C = \pi d$. The circumference is also proportional to the radius of the circle, and the constant of proportionality is $2 \cdot \pi$ because the diameter is twice as long as the radius, so $C = 2\pi r$. However, the **area of a circle** is *not* proportional to the diameter (or the radius).

The area of a circle can be found by taking the product of half the circumference and the radius. If A is the area of the circle, this gives the equation:

$$A = \frac{1}{2}(2\pi r) \cdot r$$

This equation can be rewritten as:

$$A = \pi r^2$$

(Remember that when we have $r \cdot r$ we can write r^2 and we can say "r **squared**.")

This means that if we know the radius, we can find the area. For example, if a circle has radius 10 cm, then the area is about $(3.14) \cdot 100$ which is 314 cm^2.

If we know the diameter, we can figure out the radius, and then we can find the area. For example, if a circle has a diameter of 30 ft, then the radius is 15 ft, and the area is about $(3.14) \cdot 225$ which is approximately 707 ft^2.