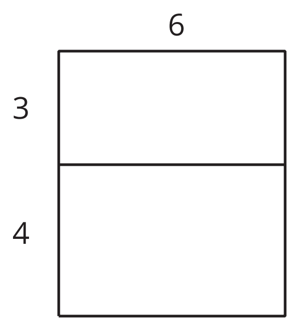
Lesson 8: Equivalent Quadratic Expressions

- Let's use diagrams to help us rewrite quadratic expressions.

8.1: Diagrams of Products

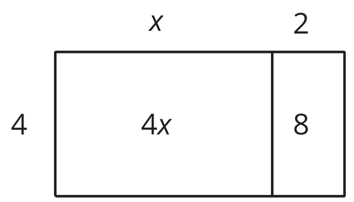


1. Explain why the diagram shows that $6(3+4) = 6 \cdot 3 + 6 \cdot 4$.

2. Draw a diagram to show that $5(x+2) = 5x + 10$.

8.2: Drawing Diagrams to Represent More Products

Applying the distributive property to multiply out the factors of, or expand, $4(x+2)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths $(x+2)$ and 4 to illustrate the multiplication.

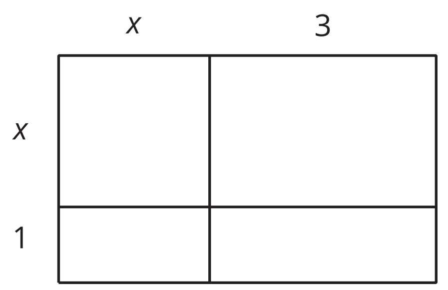


1. Draw a diagram to show that $n(2n+5)$ and $2n^2 + 5n$ are equivalent expressions.

2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.
   a. $6\left(\frac{1}{3}n+2\right)$
   b. $p(4p+9)$
   c. $5r\left(r+\frac{3}{5}\right)$
   d. $(0.5w+7)w$

8.3: Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths $x + 1$ and $x + 3$. Use this diagram to show that $(x + 1)(x + 3)$ and $x^2 + 4x + 3$ are equivalent expressions.



2. Draw diagrams to help you write an equivalent expression for each of the following:
   a. $(x + 5)^2$

   b. $2x(x + 4)$

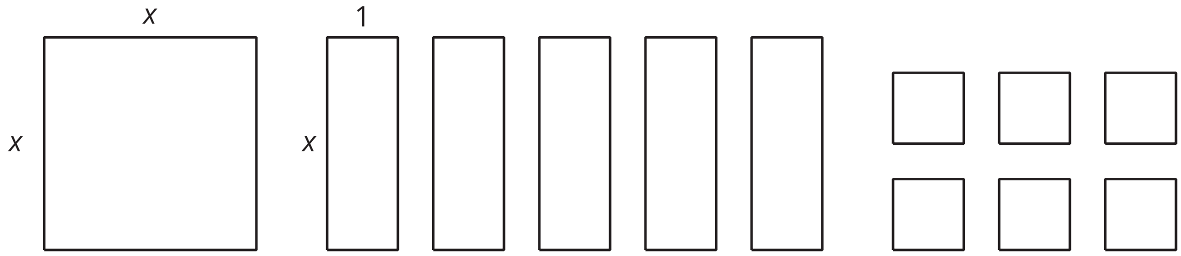
   c. $(2x + 1)(x + 3)$

   d. $(x + m)(x + n)$

3. Write an equivalent expression for each expression without drawing a diagram:
   a. $(x + 2)(x + 6)$

   b. $(x + 5)(2x + 10)$

Are you ready for more?



1. Is it possible to arrange an x by x square, five x by 1 rectangles and six 1 by 1 squares into a single large rectangle? Explain or show your reasoning.

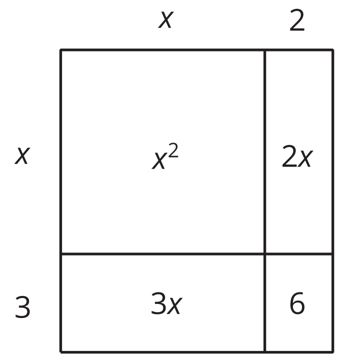
2. What does this tell you about an equivalent expression for $x^2 + 5x + 6$?

3. Is there a different non-zero number of 1 by 1 squares that we could have used instead that would allow us to arrange the combined figures into a single large rectangle?

Lesson 8 Summary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at x dollars can be expressed with $x(18 - x)$, which can also be written as $18x - x^2$. The former is a product of x and $18 - x$, and the latter is a difference of $18x$ and x^2, but both expressions represent the same function.

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $(x + 2)(x + 3)$. We can write an equivalent expression by thinking about each factor, the $(x + 2)$ and $(x + 3)$, as the side lengths of a rectangle, and each side length decomposed into a variable expression and a number.



Multiplying $(x + 2)$ and $(x + 3)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $(x + 2)(x + 3)$ is equivalent to $x^2 + 2x + 3x + 6$, or to $x^2 + 5x + 6$.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the x and the 2 in $x + 2$) is multiplied by every term in the other factor (the x and the 3 in $x + 3$).

$$(x + 2)(x + 3)$$
$$= x(x + 3) + 2(x + 3)$$
$$= x^2 + 3x + 2x + (2)(3)$$
$$= x^2 + (3+2)x + (2)(3)$$

In general, when a quadratic expression is written in the form of $(x + p)(x + q)$, we can apply the distributive property to rewrite it as $x^2 + px + qx + pq$ or $x^2 + (p + q)x + pq$.