# Lesson 17: Completing the Square and Complex Solutions

- Let's find complex solutions to quadratic equations by completing the square.

## 17.1: Creating Quadratic Equations

Match each equation in standard form to its factored form and its solutions.

1. $x^2 - 25 = 0$
2. $x^2 - 5 = 0$
3. $x^2 + 25 = 0$

- $(x - 5i)(x + 5i) = 0$
- $(x - 5)(x + 5) = 0$
- $(x - \sqrt{5})(x + \sqrt{5}) = 0$

- $\sqrt{5}$, -$\sqrt{5}$
- 5, -5
- $5i$, -$5i$

## 17.2: Sometimes the Solutions Aren't Real Numbers

What are the solutions to these equations?

1. $(x - 5)^2 = 0$

2. $(x - 5)^2 = 1$

3. $(x - 5)^2 = -1$

## 17.3: Finding Complex Solutions

Solve these equations by completing the square.

1. $x^2 - 8x + 13 = 0$

2. $x^2 - 8x + 19 = 0$

### Are you ready for more?

For which values of $a$ does the equation $x^2 - 8x + a = 0$ have two real solutions? One real solution? No real solutions? Explain your reasoning.

## 17.4: Can You See the Solutions on a Graph?

1. How many real solutions does each equation have? How many non-real solutions?
    a. $x^2 - 8x + 13 = 0$

    b. $x^2 - 8x + 16 = 0$

    c. $x^2 - 8x + 19 = 0$

2. How do the graphs of these functions help us answer the previous question?
    a. $f(x) = x^2 - 8x + 13$

    b. $g(x) = x^2 - 8x + 16$

    c. $h(x) = x^2 - 8x + 19$

## Lesson 17 Summary

Sometimes quadratic equations have real solutions, and sometimes they do not. Here is a quadratic equation with $x^2$ equal to a negative number (assume $k$ is positive):

$$x^2 = \text{-}k$$

This equation will have imaginary solutions $i\sqrt{k}$ and $\text{-}i\sqrt{k}$. By similar reasoning, an equation of the form:

$$(x-h)^2 = \text{-}k$$

will have non-real solutions if $k$ is positive. In this case, the solutions are $h + i\sqrt{k}$ and $h - i\sqrt{k}$.

It isn't always clear just by looking at a quadratic equation whether the solutions will be real or not. For example, look at this quadratic equation:

$$x^2 - 12x + 41 = 0$$

We can always complete the square to find out what the solutions will be:

$$\begin{aligned} x^2 - 12x + 36 + 5 &= 0 \\ (x-6)^2 + 5 &= 0 \\ (x-6)^2 &= \text{-}5 \\ x - 6 &= \pm i\sqrt{5} \\ x &= 6 \pm i\sqrt{5} \end{aligned}$$

This equation has non-real, complex solutions $6 + i\sqrt{5}$ and $6 - i\sqrt{5}$.