Lesson 14: More Arithmetic with Complex Numbers

- Let's practice adding, subtracting, and multiplying complex numbers.

14.1: Which One Doesn't Belong: Complex Expressions

Which one doesn't belong?

A. i^2

B. $(1+i)+(1-i)$

C. $(1+i)^2$

D. $(1+i)(1-i)$

14.2: Powers of i

1. Write each power of i in the form $a+bi$, where a and b are real numbers. If a or b is zero, you can ignore that part of the number. For example, $0+3i$ can simply be expressed as $3i$.

   i^0

   i^1

   i^2

   i^3

   i^4

   i^5

   i^6

   i^7

   i^8

2. What is i^{100}? Explain your reasoning.

3. What is i^{38}? Explain your reasoning.

Are you ready for more?

1. Write each power of $1 + i$ in the form $a + bi$, where a and b are real numbers. If a or b is zero, you can ignore that part of the number. For example, $0 + 3i$ can simply be expressed as $3i$.

   a. $(1 + i)^0$

   b. $(1 + i)^1$

   c. $(1 + i)^2$

   d. $(1 + i)^3$

   e. $(1 + i)^4$

   f. $(1 + i)^5$

   g. $(1 + i)^6$

   h. $(1 + i)^7$

   i. $(1 + i)^8$

2. Compare and contrast the powers of $1 + i$ with the powers of i. What is the same? What is different?

14.3: Add 'Em Up (or Subtract or Multiply)

For each row, your partner and you will each rewrite an expression so it has the form $a + bi$, where a and b are real numbers. You and your partner should get the same answer. If you disagree, work to reach agreement.

partner A	partner B
$(7+9i)+(3-4i)$	$5i(1-2i)$
$2i(3+4i)$	$(1+2i)-(9-4i)$
$(4-3i)(4+3i)$	$(5+i)+(20-i)$
$(2i)^4$	$(3+i\sqrt{7})(3-i\sqrt{7})$
$(1+i\sqrt{5})-(-7-i\sqrt{5})$	$(-2i)(-\sqrt{5}+4i)$
$\left(\frac{1}{2}i\right)\left(\frac{1}{3}i\right)\left(\frac{3}{4}i\right)$	$\left(\frac{1}{2}i\right)^3$

Lesson 14 Summary

Suppose we want to write the product $(3+5i)(7-2i)$ in the form $a+bi$, where a and b are real numbers. For example, we might want to compare our solution with a partner's, and having answers in the same form makes that easier. Using the distributive property,

$$\begin{aligned}(3+5i)(7-2i) &= 21-6i+35i-10i^2\\ &= 21+29i-10(\text{-}1)\\ &= 21+29i+10\\ &= 31+29i\end{aligned}$$

Keeping track of the negative signs is especially important since it is easy to mix up the fact that $i^2 = \text{-}1$ with the fact that $\text{-}i^2 = \text{-}(\text{-}1) = 1$.

Next, suppose we want to write the difference $(\text{-}6+3i)-(2-4i)$ as a single complex number in the form $a+bi$. Distributing the negative and combining like terms, we get:

$$\begin{aligned}(\text{-}6+3i)-(2-4i) &= \text{-}6+3i-2-(\text{-}4i)\\ &= \text{-}8+3i+4i\\ &= \text{-}8+7i\end{aligned}$$

Again, it is important to be precise with negative signs. It is a common mistake to just subtract $4i$ rather than subtracting $\text{-}4i$.