

Lesson 5: Negative Exponents with Powers of 10

Let's see what happens when exponents are negative.

5.1: Number Talk: What's That Exponent?

Solve each equation mentally.

$$\frac{100}{1} = 10^x$$

$$\frac{100}{x} = 10^1$$

$$\frac{x}{100} = 10^0$$

$$\frac{100}{1000} = 10^x$$

5.2: Negative Exponent Table

Complete the table to explore what negative exponents mean.

		$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$
using exponents	10^3	10^2	10^1				
as a decimal	1000.0			1.0		0.01	
as a fraction		$\frac{100}{1}$		$\frac{1}{1}$			$\frac{1}{1000}$
		$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$

- As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?
- How does a multiplier of 10 affect the placement of the decimal in the product? How does the other multiplier affect the placement of the decimal in the product?
- Use the patterns you found in the table to write 10^{-7} as a fraction.
- Use the patterns you found in the table to write 10^{-5} as a decimal.
- Write $\frac{1}{100,000,000}$ using a single exponent.
- Use the patterns in the table to write 10^{-n} as a fraction.

5.3: Follow the Exponent Rules

1. a. Match each exponential expression with an equivalent multiplication expression:

$$(10^2)^3$$

$$(10^2)^{-3}$$

$$(10^{-2})^3$$

$$(10^{-2})^{-3}$$

$\frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)}$
$(\frac{1}{10} \cdot \frac{1}{10}) (\frac{1}{10} \cdot \frac{1}{10}) (\frac{1}{10} \cdot \frac{1}{10})$
$\frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}}$
$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$

- b. Write $(10^2)^{-3}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

2. a. Match each exponential expression with an equivalent multiplication expression:

$$\frac{10^2}{10^5}$$

$$\frac{10^2}{10^{-5}}$$

$$\frac{10^{-2}}{10^5}$$

$$\frac{10^{-2}}{10^{-5}}$$

$\frac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$
$\frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{\frac{1}{10} \cdot \frac{1}{10}}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{10 \cdot 10}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$

- b. Write $\frac{10^{-2}}{10^{-5}}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

3. a. Match each exponential expression with an equivalent multiplication expression:

$10^4 \cdot 10^3$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)$

$10^4 \cdot 10^{-3}$

$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)$

$10^{-4} \cdot 10^3$

$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot (10 \cdot 10 \cdot 10)$

$10^{-4} \cdot 10^{-3}$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

- b. Write $10^{-4} \cdot 10^3$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

Are you ready for more?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game.

Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, "I knew I'd be the only one left, since I went first."

- Record this game on paper a few times with different numbers of players. Does the person who starts always win?
- Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

Lesson 5 Summary

When we multiply a positive power of 10 by $\frac{1}{10}$, the exponent *decreases* by 1:

$$10^8 \cdot \frac{1}{10} = 10^7$$

This is true for *any* positive power of 10. We can reason in a similar way that multiplying by 2 factors that are $\frac{1}{10}$ *decreases* the exponent by 2:

$$\left(\frac{1}{10}\right)^2 \cdot 10^8 = 10^6$$

That means we can extend the rules to use negative exponents if we make $10^{-2} = \left(\frac{1}{10}\right)^2$. Just as 10^2 is two factors that are 10, we have that 10^{-2} is two factors that are $\frac{1}{10}$. More generally, the exponent rules we have developed are true for *any* integers n and m if we make

$$10^{-n} = \left(\frac{1}{10}\right)^n = \frac{1}{10^n}$$

Here is an example of extending the rule $\frac{10^n}{10^m} = 10^{n-m}$ to use negative exponents:

$$\frac{10^3}{10^5} = 10^{3-5} = 10^{-2}$$

To see why, notice that

$$\frac{10^3}{10^5} = \frac{10^3}{10^3 \cdot 10^2} = \frac{10^3}{10^3} \cdot \frac{1}{10^2} = \frac{1}{10^2}$$

which is equal to 10^{-2} .

Here is an example of extending the rule $(10^m)^n = 10^{m \cdot n}$ to use negative exponents:

$$(10^{-2})^3 = 10^{(-2)(3)} = 10^{-6}$$

To see why, notice that $10^{-2} = \frac{1}{10} \cdot \frac{1}{10}$. This means that

$$(10^{-2})^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right)^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) = \frac{1}{10^6} = 10^{-6}$$