## Lesson 7: Expressing Transformations of Functions Algebraically

* Let’s express transformed functions algebraically.

### 7.1: Describing Translations

Let $g\left(x\right)=\sqrt{x}$. Complete the table. Be prepared to explain your reasoning.

| words (the graph of $y=g\left(x\right)$ is...) | function notation | expression |
| --- | --- | --- |
| translated left 5 units | $g\left(x+5\right)$ |   |
| translated left 5 units and down 3 units |   | $\sqrt{x+5}−3$ |
|   | $g\left(-x\right)$ | $\sqrt{-x}$ |
| translated left 5 units, then down 3 units,then reflected across the $y$-axis |   |   |

### 7.2: Translating Vertex Form

Let $f$ be the function given by $f\left(x\right)=x^{2}$.

1. Write an equation for the function $g$ whose graph is the graph of $f$ translated 3 units left and up 5 units.
2. What is the vertex of the graph of $g$? Explain how you know.
3. Write an equation for a quadratic function $h$ whose graph has a vertex at $\left(1.5,2.6\right)$.
4. Write an equation for a quadratic function $k$ whose graph opens downward and has a vertex at $\left(3.2,-4.7\right)$.

### 7.3: An Even Better Fit

In an earlier lesson, we looked at the temperature $T$, in degrees Fahrenheit, of a bottle of soda water left outside for $h$ hours. Let’s model this data with a function. This time, we will start with the function $f\left(h\right)=33\left(0.6\right)^{h}$. This graph has a shape that fits the data well.





1. Describe a translation of this graph that fits the data.
2. Write an equation defining a function $g$ that models the data.
3. What does your function tell you about the temperature outside?

#### Are you ready for more?

Han tried the following steps to model the soda water temperature. First he shifts the given graph left by one hour, then he applies a vertical shift.

1. What vertical shift does Han need to apply to model the 45 degree Fahrenheit temperature in the refrigerator?
2. How does Han’s model compare to yours?

### Lesson 7 Summary

You can use the equation of a function to write an equation for its transformation. For example, let $f\left(x\right)=x^{2}$. Take the graph of $f$, reflect it across the $x$-axis, translate it up 10 units, and translate it left 3 units. What is an equation for this new function? The new function $g$ is related to $f$ by $g\left(x\right)=-f\left(x+3\right)+10$, since



Which means $g\left(x\right)=-\left(x+3\right)^{2}+10$.

Sometimes you can recognize from the expression for a function that it is the transformation of a simpler function. For example, consider:

$H\left(t\right)=10−\left(1.2\right)^{t+5}$

One way to obtain the expression for $H$ from $1.2^{t}$ is:

* adding 5 to the input to get $\left(1.2\right)^{t+5}$
* multiplying the output by -1 to get $-\left(1.2\right)^{t+5}$
* adding 10 to the output to get $10−\left(1.2\right)^{t+5}$

So the graph of $H$ is obtained from the graph of $f\left(t\right)=1.2^{t}$ by translating left 5 units, reflecting across the $x$-axis, and translating up 10 units. Consider the point $\left(0,1\right)$ on the graph of $f$. After translating, reflecting, and translating again, it becomes the point $\left(-5,9\right)$ on the graph of $H$.





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