## Lesson 4: Reflecting Functions

* Let’s reflect some graphs.

### 4.1: Notice and Wonder: Reflections

What do you notice? What do you wonder?







### 4.2: Reflecting Across

Here is the graph of function $f$ and a table of values.



| $x$ | $f\left(x\right)$ | $g\left(x\right)=-f\left(x\right)$ |
| --- | --- | --- |
| -3 | 0 |   |
| -1.5 | -4.3 |   |
| -1 | -4 |   |
| 0 | -1.8 |   |
| 0.6 | 0 |   |
| 2.6 | 3.9 |   |
| 4 | 0 |   |

1. Let $g$ be the function defined by $g\left(x\right)=-f\left(x\right)$. Complete the table.
2. Sketch the graph of $g$ on the same axes as the graph of $f$ but in a different color.
3. Describe how to transform the graph of $f$ into the graph of $g$. Explain how the equation produces this transformation.

### 4.3: Reflecting Across a Different Way

Here is another copy of the graph of $f$ from the earlier activity. This time, let $h$ be the function defined by $h\left(x\right)=f\left(-x\right)$.



1. Use the definition of $h$ to find $h\left(0\right)$. Does your answer agree with your prediction?
2. What does your prediction tell you about $h\left(-0.6\right)$? Does your answer agree with the definition of $h$?
3. Complete the tables. The values for $x$ will not be the same for the two tables.

| * $x$
 | * $f\left(x\right)$
 |
| --- | --- |
| * -3
 | * 0
 |
| * -1.5
 | * -4.3
 |
| * -1
 | * -4
 |
| * 0
 | * -1.8
 |
| * 0.6
 | * 0
 |
| * 2.6
 | * 3.9
 |
| * 4
 | * 0
 |

| * $x$
 | * $h\left(x\right)=f\left(-x\right)$
 |
| --- | --- |
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1. Sketch the graph of $h$ on the same axes as the graph of $f$ but in a different color.
2. Describe what happened to the graph of $f$ to transform it into the graph of $h$. Explain how the equation produces this transformation.

#### Are you ready for more?

1. Describe how the graph of $h$ relates to the graph of $g$ defined in the earlier activity.
2. Write an equation relating $h$ and $g$.

### Lesson 4 Summary

Here are graphs of the functions $f$, $g$, and $h$, where $g\left(x\right)=-f\left(x\right)$ and $h\left(x\right)=f\left(-x\right)$. How do these equations match the transformation we see from $f$ to $g$ and from $f$ to $h$?

$f\left(x\right)$



$g\left(x\right)=-f\left(x\right)$



$h\left(x\right)=f\left(-x\right)$



Considering first the equation $g\left(x\right)=-f\left(x\right)$, we know that for the same input $x$, the value of $g\left(x\right)$ will be the opposite of the value of $f\left(x\right)$. For example, since $f\left(0\right)=1$, we know that $g\left(0\right)=-f\left(0\right)=-1$. We can see this relationship in the graphs where $g$ is the reflection of $f$ across the $x$-axis.

Looking at $h\left(x\right)=f\left(-x\right)$, this equation tells us that the two functions have the same output for opposite inputs. For example, 1 and -1 are opposites, so $h\left(1\right)=f\left(-1\right)$ (and $h\left(-1\right)=f\left(1\right)$ is also true!). We can see this relationship in the graphs where $h$ is the reflection of $f$ across the $y$-axis.



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