

Lesson 21: Odd and Even Numbers

- Let's explore even and odd numbers.

21.1: Math Talk: Evens and Odds

Evaluate mentally.

$$64 + 88$$

$$65 + 89$$

$$14 \cdot 5$$

$$14 \cdot 4$$

21.2: Always Even, Never Odd

Here are some statements about the sums and products of numbers. For each statement:

- decide whether it is *always* true, true for *some* numbers but not others, or *never* true
- use examples to explain your reasoning

1. Sums:

- The sum of 2 even numbers is even.
- The sum of an even number and an odd number is odd.
- The sum of 2 odd numbers is odd.

2. Products:

- The product of 2 even numbers is even.
- The product of an even number and an odd number is odd.

c. The product of 2 odd numbers is odd.

21.3: Even + Odd = Odd

How do we know that the sum of an even number and an odd number *must* be odd? Examine this proof and answer the questions throughout.

Let a represent an even number, b represent an odd number, and s represent the sum $a + b$.

1. What does it mean for a number to be even? Odd?

Assume that s is even, then we will look for a reason the original statement cannot be true. Since a and s are even, we can write them as 2 times an integer. Let $a = 2k$ and $s = 2m$ for some integers k and m .

2. Can this always be done? To convince yourself, write 4 different even numbers. What is the value for k for each of your numbers when you set them equal to $2k$?

Then we know that $a + b = s$ and $2k + b = 2m$.

Divide both sides by 2 to get that $k + \frac{b}{2} = m$.

Rewrite the equation to get $\frac{b}{2} = m - k$.

Since m and k are integers, then $\frac{b}{2}$ must be an integer as well.

3. Is the difference of 2 integers always an integer? Select 4 pairs of integers and subtract them to convince yourself that their difference is always an integer.

4. What does the equation $\frac{b}{2} = m - k$ tell us about $\frac{b}{2}$? What does that mean about b ?

5. Look back at the original description of b . What is wrong with what we have discovered?

The logic for everything in the proof works, so the only thing that could've gone wrong was our assumption that s is even. Therefore, s must be odd.