## Lesson 7: Angle-Side-Angle Triangle Congruence

* Let’s see if we can prove other sets of measurements that guarantee triangles are congruent, and apply those theorems.

### 7.1: Notice and Wonder: Assertion

Assertion: Through 2 distinct points passes a unique line. Two lines are said to be *distinct* if there is at least 1 point that belongs to one but not the other. Otherwise, we say the lines are the same. Lines that have no point in common are said to be *parallel*.

Therefore, we can conclude: given 2 distinct lines, either they are parallel, or they have exactly 1 point in common.

What do you notice? What do you wonder?

### 7.2: Proving the Angle-Side-Angle Triangle Congruence Theorem

1. Two triangles have 2 pairs of corresponding angles congruent, and the corresponding sides between those angles are congruent. Sketch 2 triangles that fit this description.
2. Label the triangles $WXY$ and $DEF$, so that angle $W$ is congruent to angle $D$, angle $X$ is congruent to angle $E$, and side $WX$ is congruent to side $DE$.
3. Use a sequence of rigid motions to take triangle $WXY$ onto triangle $DEF$. For each step, explain how you know that one or more vertices will line up.

### 7.3: Find the Missing Angle Measures

Lines $ℓ$ and $m$ are parallel. $a=42$. Find $b$, $c$, $d$, $e$, $f$, $g$, and $h$.

$ℓ∥m$



### 7.4: What Do We Know For Sure About Parallelograms?

Quadrilateral $ABCD$ is a **parallelogram**. By definition, that means that segment $AB$ is parallel to segment $CD$, and segment $BC$ is parallel to segment $AD$.

1. Sketch parallelogram $ABCD$ and then draw an auxiliary line to show how $ABCD$ can be decomposed into 2 triangles.
2. Prove that the 2 triangles you created are congruent, and explain why that shows one pair of opposite sides of a parallelogram must be congruent.

#### Are you ready for more?

When we have 3 consecutive vertices of a polygon $A$, $B$, and $C$ so that the triangle $ABC$ lies entirely inside the polygon, we call $B$ an *ear* of the polygon.

1. How many ears does a parallelogram have?
2. Draw a quadrilateral that has fewer ears than a parallelogram.
3. In 1975, Gary Meisters proved that every polygon has at least 2 ears. Draw a hexagon with only 2 ears.

### Lesson 7 Summary

We know that in 2 triangles, if 2 pairs of corresponding sides and the pair of corresponding angles between the sides are congruent, then the triangles must be congruent. But we don’t always know that 2 pairs of corresponding sides are congruent. For example, when proving that opposite sides are congruent in any parallelogram, we only have information about 1 pair of corresponding sides. That is why we need other ways than the Side-Angle-Side Triangle Congruence Theorem to prove triangles are congruent.

In 2 triangles, if 2 pairs of corresponding angles and the pair of corresponding sides between the angles are congruent, then the triangles must be congruent. This is called the *Angle-Side-Angle Triangle Congruence Theorem*.



When proving that 2 triangles are congruent, look at the diagram and given information and think about whether it will be easier to find 2 pairs of corresponding angles that are congruent or 2 pairs of corresponding sides that are congruent. Then check if there is enough information to use the Angle-Side-Angle Triangle Congruence Theorem or the Side-Angle-Side Triangle Congruence Theorem.

The Angle-Side-Angle Triangle Congruence Theorem can be used to prove that, in a **parallelogram**, opposite sides are congruent. A parallelogram is defined to be a quadrilateral with 2 pairs of opposite sides parallel.



We could prove that triangles $ABC$ and $CDA$ are congruent by the Angle-Side-Angle Triangle Congruence Theorem. Then we can say segment $AD$ is congruent to segment $CB$ because they are corresponding parts of congruent triangles.



© CC BY 2019 by Illustrative Mathematics®