Lesson 9: What is a Logarithm?

- Let’s learn about logarithms.

9.1: Math Talk: Finding Solutions

Find or estimate the value of each variable mentally.

$4^a = 16$

$4^b = 2$

$4^{\frac{5}{2}} = c$

$4^d = 56$

9.2: A Table of Numbers

x	$\log_{10}(x)$	x	$\log_{10}(x)$	x	$\log_{10}(x)$	x	$\log_{10}(x)$
2	0.3010	20	1.3010	200	2.3010	2,000	3.3010
3	0.4771	30	1.4771	300	2.4771	3,000	3.4771
4	0.6021	40	1.6021	400	2.6021	4,000	3.6021
5	0.6990	50	1.6990	500	2.6990	5,000	3.6990
6	0.7782	60	1.7782	600	2.7782	6,000	3.7782
7	0.8451	70	1.8451	700	2.8451	7,000	3.8451
8	0.9031	80	1.9031	800	2.9031	8,000	3.9031
9	0.9542	90	1.9542	900	2.9542	9,000	3.9542
10	1	100	2	1,000	3	10,000	4

1. Analyze the table and discuss with a partner what you think the table tells us.

2. Use the table to find the value of the unknown exponent that makes each equation true.
    a. $10^w = 1{,}000$
    b. $10^y = 9$
    c. $10^z = 90$

3. Notice that some values in the columns labeled $\log_{10} x$ are whole numbers, but most are decimals. Why do you think that is?

9.3: Hello, Logarithm!

1. Here are two true equations based on the information from the table:

$$\log_{10} 100 = 2$$
$$\log_{10} 1{,}000 = 3$$

   What values could replace the "?" in these equations to make them true?

   a. $\log_{10} 1{,}000{,}000 = ?$

   b. $\log_{10} 1 = ?$

   c. $\log_{10} ? = 4$

2. Between which two whole numbers is the value of $\log_{10} 600$? Be prepared to explain how do you know.

3. The term *log* is short for **logarithm**. Discuss the following questions with a partner and record your responses:
   a. What do you think logarithm means or does?

   b. Next to "log" is a subscript—a number or letter printed smaller and below the line of text. What do you think the subscript tells us?

   c. What about the other two numbers on either side of the equal sign (for example, the 100 and the 2 in $\log_{10} 100 = 2$)? What do they tell us?

Are you ready for more?

1. For which whole number values of n is $\log_{10}(n)$ an integer?

2. Why will $\log_{10}(n)$ never be equal to a non-integer rational number?

Lesson 9 Summary

We know how to solve equations such as $10^a = 10{,}000$ or $10^b = \frac{1}{100}$ by thinking about integer powers of 10. The solutions are $a = 4$ and $b = -2$. What about an equation such as $10^p = 250$?

Because $10^2 = 100$ and $10^3 = 1{,}000$, we know that p is between 2 and 3. We can use a **logarithm** to represent the exact solution to this equation and write it as:

$$p = \log_{10} 250$$

The expression is read "the log, base 10, of 250."

- The small, slightly lowered "10" refers to the base of 10.
- The 250 is the value of the power of 10.
- $\log_{10} 250$ is the value of the exponent p that makes 10^p equal 250.

Base 10 logarithms are often written without the number 10. So $\log_{10} 250$ can also be written as $\log 250$ and this expression is read "the log of 250."

One way to estimate logarithms is with a logarithm table. For example, using this base 10 logarithm table we can see that $\log_{10} 250$ is between 2.3 and 2.48.

x	$\log_{10}(x)$
2	0.3010
3	0.4771
4	0.6021
5	0.6990
6	0.7782
7	0.8451
8	0.9031
9	0.9542
10	1

x	$\log_{10}(x)$
20	1.3010
30	1.4771
40	1.6021
50	1.6990
60	1.7782
70	1.8451
80	1.9031
90	1.9542
100	2

x	$\log_{10}(x)$
200	2.3010
300	2.4771
400	2.6021
500	2.6990
600	2.7782
700	2.8451
800	2.9031
900	2.9542
1,000	3

x	$\log_{10}(x)$
2,000	3.3010
3,000	3.4771
4,000	3.6021
5,000	3.6990
6,000	3.7782
7,000	3.8451
8,000	3.9031
9,000	3.9542
10,000	4