## Lesson 5: Negative Rational Exponents

* Let’s investigate negative exponents.

### 5.1: Math Talk: Don’t Be Negative

Evaluate mentally.

$9^{2}$

$9^{-2}$

$9^{\frac{1}{2}}$

$9^{-\frac{1}{2}}$

### 5.2: Negative Fractional Powers Are Just Numbers

1. Complete the table as much as you can without using a calculator. (You should be able to fill in three spaces.)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| * $x$
 | * -2
 | * $-\frac{5}{3}$
 | * $-\frac{4}{3}$
 | * -1
 | * $-\frac{2}{3}$
 | * $-\frac{1}{3}$
 | * 0
 |
| * $2^{x}$ (using exponents)
 | * $2^{-2}$
 | * $2^{-\frac{5}{3}}$
 | * $2^{-\frac{4}{3}}$
 | * $2^{-1}$
 | * $2^{-\frac{2}{3}}$
 | * $2^{-\frac{1}{3}}$
 | * $2^{0}$
 |
| * $2^{x}$ (decimal approximation)
 |  |  |  |  |  |  |  |

* 1. Plot these powers of 2 in the coordinate plane. ​​​​​​
	2. Connect the points as smoothly as you can.
	3. Use your graph of $y=2^{x}$ to estimate the value of the other powers in the table, and write your estimates in the table.
* 
1. Let’s investigate $2^{-\frac{1}{3}}$.
	1. Write $2^{-\frac{1}{3}}$ using radical notation.
	2. What is the value of $\left(2^{-\frac{1}{3}}\right)^{3}$?
	3. Raise your estimate of $2^{-\frac{1}{3}}$ to the third power. What should it be? How close did you get?
2. Let’s investigate $2^{-\frac{2}{3}}$.
	1. Write $2^{-\frac{2}{3}}$ using radical notation.
	2. What is $\left(2^{-\frac{2}{3}}\right)^{3}$?
	3. Raise your estimate of $2^{-\frac{2}{3}}$ to the third power. What should it be? How close did you get?

### 5.3: Any Fraction Can Be an Exponent

1. For each set of 3 numbers, cross out the expression that is not equal to the other two expressions.
	1. $8^{\frac{4}{5}}$, $\sqrt[4]{8}^{5}$, $\sqrt[5]{8}^{4}$
	2. $8^{-\frac{4}{5}}$, $\frac{1}{\sqrt[5]{8^{4}}}$, $-\frac{1}{\sqrt[5]{8^{4}}}$
	3. $\sqrt{4^{3}}$, $4^{\frac{3}{2}}$, $4^{\frac{2}{3}}$
	4. $\frac{1}{\sqrt{4^{3}}}$, $-4^{\frac{3}{2}}$, $4^{-\frac{3}{2}}$
2. For each expression, write an equivalent expression using radicals.
	1. $17^{\frac{3}{2}}$
	2. $31^{-\frac{3}{2}}$
3. For each expression, write an equivalent expression using only exponents.
	1. $\left(\sqrt{3}\right)^{4}$
	2. $\frac{1}{\left(\sqrt[3]{5}\right)^{6}}$

#### Are you ready for more?

Write two different expressions that involve only roots and powers of 2 which are equivalent to $\frac{4^{\frac{2}{3}}}{8^{\frac{1}{4}}}$.

### 5.4: Make These Exponents Less Complicated

Match expressions into groups according to whether they are equal. Be prepared to explain your reasoning.

$\left(\sqrt{3}\right)^{4}$

$\sqrt{3^{2}}$

$\left(3^{\frac{1}{2}}\right)^{4}$

$\left(\sqrt{3}\right)^{2}⋅\left(\sqrt{3}\right)^{2}$

$\left(3^{2}\right)^{\frac{1}{2}}$

$3^{2}$

$3^{\frac{4}{2}}$

$\left(3^{\frac{1}{2}}\right)^{2}$

### Lesson 5 Summary

When we have a number with a negative exponent, it just means we need to find the reciprocal of the number with the exponent that has the same magnitude, but is positive. Here are two examples:

$7^{-5}=\frac{1}{7^{5}}$

$7^{-\frac{6}{5}}=\frac{1}{7^{\frac{6}{5}}}$

The table shows a few more examples of exponents that are fractions and their radical equivalents.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $x$ | -1 | $-\frac{2}{3}$ | $-\frac{1}{3}$ | 0 | $\frac{1}{3}$ | $\frac{2}{3}$ | 1 |
| $5^{x}$ (using exponents) | $5^{-1}$ | $5^{-\frac{2}{3}}$ | $5^{-\frac{1}{3}}$ | $5^{0}$ | $5^{\frac{1}{3}}$ | $5^{\frac{2}{3}}$ | $5^{1}$ |
| $5^{x}$ (equivalent expressions) | $\frac{1}{5}$ | $\frac{1}{\sqrt[3]{5^{2}}}$ or $\frac{1}{\sqrt[3]{25}}$ | $\frac{1}{\sqrt[3]{5}}$ | 1 | $\sqrt[3]{5}$ | $\sqrt[3]{5^{2}}$ or $\sqrt[3]{25}$ | 5 |



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