## Lesson 7: Inequivalent Equations

* Let’s see what happens when we square each side of an equation.

### 7.1: 2 and -2

What do you notice? What do you wonder?

* $x^{2}=4$
* $x^{2}=-4$
* $\left(x−2\right)\left(x+2\right)=0$
* $x=\sqrt{4}$

### 7.2: Careful When You Take the Square Root

Tyler was solving this equation:

$x^{2}−1=3$

He said, “I can add 1 to each side of the equation and it doesn’t change the equation. I get $x^{2}=4$.”

1. Priya said, “It does change the equation. It just doesn’t change the solutions!” Then she showed these two graphs.
* Figure A
* 
* Figure B
* 
	1. How can you see the solution to the equation $x^{2}−1=3$ in Figure A?
	2. How can you see the solution to the equation $x^{2}=4$ in Figure B?
	3. Use the graphs to explain why the equations have the same solutions.
1. Tyler said, “Now I can take the square root of each side to get the solution to $x^{2}=4$. The square root of $x^{2}$ is $x$. The square root of 4 is 2.” He wrote:
* $\begin{matrix}x^{2}&=4\\\sqrt{x^{2}}&=\sqrt{4}\\x&=2\end{matrix}$
* Priya said, “But the graphs show that there are *two* solutions!” What went wrong?

### 7.3: Another Way to Solve

Han was solving this equation: $\frac{x+3}{2}=4$

He said, "I know that half of $x+3$ is 4. So $x+3$ must be 8, since half of 8 is 4. This means that $x$ is 5."

1. Use Han's reasoning to solve this equation: $\left(x+3\right)^{2}=4$.
2. What advice would you give to someone who was going to solve an equation like $\left(x+3\right)^{2}=4$?

### 7.4: What Happens When You Square Each Side?

Mai was solving this equation: $\sqrt{x−1}=3$

She said, “I can square each side of the equation to get another equation with the same solutions.” Then she wrote:

$\begin{matrix}\sqrt{x−1}&=3\\\left(\sqrt{x−1}\right)^{2}&=3^{2}\\x−1&=9\\x&=10\end{matrix}$

1. Check to see if her solution makes the original equation true.
2. Andre said, “I tried your technique to solve $\sqrt{x−1}=-3$but it didn’t work.” Why doesn’t it work? Explain or show your reasoning.

### 7.5: Solve These Equations With Square Roots in Them

Find the solution(s) to each of these equations, or explain why there is no solution.

1. $\sqrt{t+4}=3$
2. $-10=-\sqrt{a}$
3. $\sqrt{3−w}−4=0$
4. $\sqrt{z}+9=0$

#### Are you ready for more?

Are there values of $a$ and $b$ so that the equation $\sqrt{t+a}=b$ has more than one solution? Explain your reasoning.

### Lesson 7 Summary

Every positive number has *two* square roots. You can see this by looking at the graph of $y=x^{2}$:



If $y$ is a positive number like 4, then we can see that $y=4$ crosses the graph in two places, so the equation $x^{2}=4$ will have two solutions, namely, $\sqrt{4}$ and $-\sqrt{4}$. This is true for any positive number $a$: $y=a$ will cross the graph in two places, and $x^{2}=a$ will have two solutions, $x=\sqrt{a}$ and $x=-\sqrt{a}$.

When we have a square root in an equation like $\sqrt{t}−6=0$, we can isolate the square root and then square each side:

$\begin{matrix}\sqrt{t}−6&=0\\\sqrt{t}&=6\\t&=6^{2}\\t&=36\end{matrix}$

But sometimes, squaring each side of an equation gives results that aren’t solutions to the original equation. For example:

$\begin{matrix}\sqrt{t}+6&=0\\\sqrt{t}&=-6\\t&=\left(-6\right)^{2}\\t&=36\end{matrix}$

Note that 36 is *not* a solution to the original equation, because $\sqrt{36}+6$ doesn’t equal 0. In fact, $\sqrt{t}+6=0$ has no solutions, because it’s impossible for the sum of two positive numbers to be zero.

Remember: sometimes the new equation has solutions that the old equation doesn’t have. Always check your solutions in the original equation!



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