Lesson 10: Interpreting and Writing Logarithmic Equations

- Let’s look at logarithms with different bases.

10.1: Reading Logs

The expression $\log_{10} 1{,}000 = 3$ can be read as: “The log, base 10, of 1,000 is 3.”

It can be interpreted as: “The exponent to which we raise a base 10 to get 1,000 is 3.”

Take turns with a partner reading each equation out loud. Then, interpret what they mean.

- $\log_{10} 100{,}000{,}000 = 8$
- $\log_{10} 1 = 0$
- $\log_2 16 = 4$
- $\log_5 25 = 2$

10.2: Base 2 Logarithms

x	$\log_2(x)$
1	0
2	1
3	1.5850
4	2
5	2.3219
6	2.5850
7	2.8074
8	3
9	3.1699
10	3.3219

x	$\log_2(x)$
11	3.4594
12	3.5845
13	3.7004
14	3.8074
15	3.9069
16	4
17	4.0875
18	4.1699
19	4.2479
20	4.3219

x	$\log_2(x)$
21	4.3923
22	4.4594
23	4.5236
24	4.5850
25	4.6439
26	4.7004
27	4.7549
28	4.8074
29	4.8580
30	4.9069

x	$\log_2(x)$
31	4.9542
32	5
33	5.0444
34	5.0875
35	5.1293
36	5.1699
37	5.2095
38	5.2479
39	5.2854
40	5.3219

1. Use the table to find the exact or approximate value of each expression. Then, explain to a partner what each expression and its approximated value means.
   a. $\log_2 2$

   b. $\log_2 32$

   c. $\log_2 15$

   d. $\log_2 40$

2. Solve each equation. Write the solution as a logarithmic expression.
   a. $2^y = 5$

   b. $2^y = 70$

   c. $2^y = 999$

10.3: Exponential and Logarithmic Forms

These equations express the same relationship between 2, 16, and 4:

$$\log_2 16 = 4 \qquad\qquad 2^4 = 16$$

1. Each row shows two equations that express the same relationship. Complete the table.

	exponential form	logarithmic form
a.	$2^1 = 2$	
b.	$10^0 = 1$	
c.		$\log_3 81 = 4$
d.		$\log_5 1 = 0$
e.	$10^{-1} = \frac{1}{10}$	
f.	$9^{\frac{1}{2}} = 3$	
g.		$\log_2 \frac{1}{8} = -3$
h.	$2^y = 15$	
i.		$\log_5 40 = y$
j.	$b^y = x$	

2. Write two equations—one in exponential form and one in logarithmic form—to represent each question. Use "?" for the unknown value.
   a. "To what exponent do we raise the number 4 to get 64?"

   b. "What is the log, base 2, of 128?"

Are you ready for more?

1. Is $\log_2(10)$ greater than 3 or less than 3? Is $\log_{10}(2)$ greater than or less than 1? Explain your reasoning.

2. How are these two quantities related?

Lesson 10 Summary

Many relationships that can be expressed with an exponent can also be expressed with a logarithm. Let's look at this equation:

$$2^7 = 128$$

The base is 2 and the exponent is 7, so it can be expressed as a logarithm with base 2:

$$\log_2 128 = 7$$

In general, an exponential equation and a logarithmic equation are related as shown here:

$$\underset{\text{base}}{b}^{\overset{\text{exponent}}{y}} = x \qquad \log_{\underset{\text{base}}{b}} x = \overset{\text{exponent}}{y}$$

Exponents can be negative, so a logarithm can have negative values. For example $3^{-4} = \frac{1}{81}$, which means that $\log_3 \frac{1}{81} = -4$.

An exponential equation cannot always be solved by observation. For example, $2^x = 19$ does not have an obvious solution. The logarithm gives us a way to represent the solution to this equation: $x = \log_2 19$. The expression $\log_2 19$ is approximately 4.25, but $\log_2 19$ is an exact solution.