Evaluating a Sample Response to a Modeling Prompt

What do people mean by *modeling with mathematics*? Here is a prompt that could be approached by modeling, and a sample response to understand and evaluate.

Prompt:

Two friends, Han and Jada, live 7 miles apart. One Saturday, they decide to meet up somewhere between their houses. They each leave their house at 8 a.m. and travel toward each other. They want to choose a place to meet so that they'll both arrive at the same time. Where could they meet?

Sample Response:

Let's assume we don't have to worry about delays like stoplights or traffic. To simplify the situation, let's pretend that they each travel at a constant speed the whole time. Here are some realistic speeds for different methods of travel:

- Walking 2.5–4 miles per hour
- Driving 25–30 miles per hour
- Biking 10–14 miles per hour

If they each use the same method of transportation, they'll be going about the same speed and will meet pretty much halfway between their houses. For example, let's say they both ride bikes, and Han goes 11 mph (miles per hour) and Jada goes 14 mph. Then here's how to tell where they'll meet:

Call the distance that Han travels "h" and the distance Jada travels "j". They both travel for the same amount of time, call it "t". Since Han goes 11 miles per hour, $\frac{h}{t}$ must equal 11. Since Jada goes 14 miles per hour, $\frac{j}{t}$ must equal 14. So we can rearrange to solve for t:

$\frac{h}{t} = 11$ becomes $t = \frac{h}{11}$

$\frac{j}{t} = 14$ becomes $t = \frac{j}{14}$

Now we know that $\frac{h}{11}$ and $\frac{j}{14}$ must equal each other, since they both equal t. But we still have two unknowns. So we need to use the information that they live 7 miles apart. That means that the distances they travel have to add up to 7 miles. So $h + j = 7$.

Now we can solve for one of the variables and use substitution to find the other one:

$h = 7 - j$

$\frac{h}{11} = \frac{j}{14}$

$\frac{7-j}{11} = \frac{j}{14}$

$7 - j = 11\left(\frac{j}{14}\right)$

$7 = j\left(\frac{11}{14}\right) + j$

$7 = j\left(\frac{11}{14} + 1\right) = j\left(\frac{25}{14}\right)$

$7 \cdot \left(\frac{14}{25}\right) = j$

So, $j = 3.92$.

So Jada travels 3.92 miles, which means Han travels $7 - 3.92 = 3.08$ miles. Since they were going close to the same speed, they met pretty close to the middle. It makes sense that Jada traveled a bit farther, since she was going faster.

If their speeds are very different, then they won't meet near the middle, and they'll meet much closer to the slower person's house. For example, if Jada drives at an average of 28 mph and Han rides his bike at 11 mph, then:

$\frac{h}{t} = 11$ becomes $t = \frac{h}{11}$

$\frac{j}{t} = 28$ becomes $t = \frac{j}{28}$

So $\frac{h}{11} = \frac{j}{28}$. Now we can do the same calculations as before:

$h = 7 - j$

$\frac{h}{11} = \frac{j}{28}$

$\frac{7-j}{11} = \frac{j}{28}$

$7 - j = 11\left(\frac{j}{28}\right)$

$$7 = j\left(\frac{11}{28}\right) + j$$

$$7 = j\left(\frac{11}{28} + 1\right) = j\left(\frac{39}{28}\right)$$

$$7 \cdot \left(\frac{28}{39}\right) = j$$

So Jada travels 5.02 miles, which means Han travels only 1.98 miles.

Data Sources

1. https://en.wikipedia.org/wiki/Walking
2. https://www.livestrong.com/article/486666-is-an-average-of-15-miles-per-hour-on-a-bike-good-for-a-beginner/
3. https://www.forbes.com/sites/jimgorzelany/2017/09/27/the-worlds-best-and-worst-cities-for-drivers/