

Lesson 12: Infinite Decimal Expansions

Let's think about infinite decimals.

12.1: Searching for Digits

The first 3 digits after the decimal for the decimal expansion of $\frac{3}{7}$ have been calculated. Find the next 4 digits.

$$\begin{array}{r}
 0.428 \\
 \hline
 7 \overline{) 3} \\
 \underline{- 28} \\
 20 \\
 \underline{- 14} \\
 60 \\
 \underline{- 56} \\
 4
 \end{array}$$

12.2: Some Numbers Are Rational

Your teacher will give your group a set of cards. Each card will have a calculations side and an explanation side.

- The cards show Noah's work calculating the fraction representation of $0.\overline{485}$. Arrange these in order to see how he figured out that $0.\overline{485} = \frac{481}{990}$ without needing a calculator.

- Use Noah's method to calculate the fraction representation of:

a. $0.\overline{186}$

b. $0.\overline{788}$

Are you ready for more?

Use this technique to find fractional representations for $0.\overline{3}$ and $0.\overline{9}$.

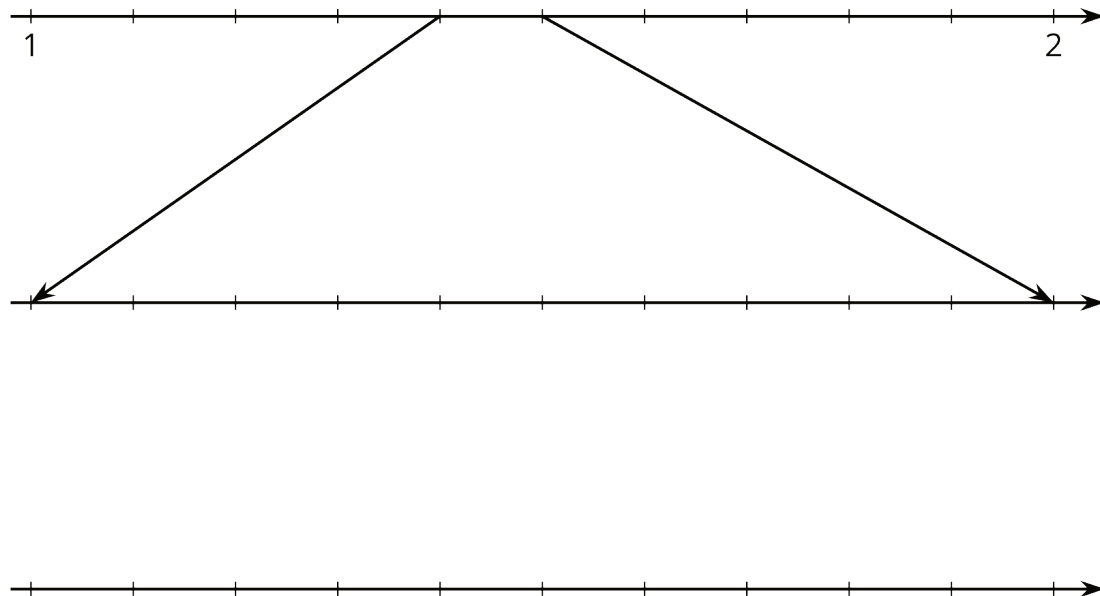
12.3: Some Numbers Are Not Rational

- Why is $\sqrt{2}$ between 1 and 2 on the number line?

b. Why is $\sqrt{2}$ between 1.4 and 1.5 on the number line?

c. How can you figure out an approximation for $\sqrt{2}$ accurate to 3 decimal places?

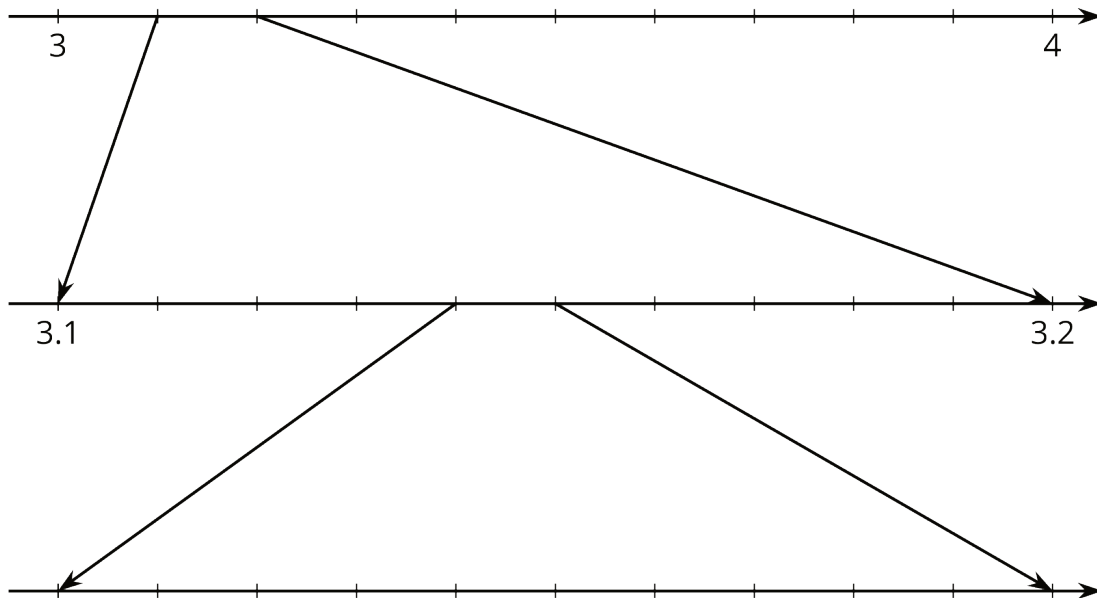
d. Label all of the tick marks. Plot $\sqrt{2}$ on all three number lines. Make sure to add arrows from the second to the third number lines.



2. a. Elena notices a beaker in science class says it has a diameter of 9 cm and measures its circumference to be 28.3 cm. What value do you get for π using these values and the equation for circumference, $C = 2\pi r$?

b. Diego learned that one of the space shuttle fuel tanks had a diameter of 840 cm and a circumference of 2,639 cm. What value do you get for π using these values and the equation for circumference, $C = 2\pi r$?

c. Label all of the tick marks on the number lines. Use a calculator to get a very accurate approximation of π and plot that number on all three number lines.



d. How can you explain the differences between these calculations of π ?

Lesson 12 Summary

Not every number is rational. Earlier we tried to find a fraction whose square is equal to 2. That turns out to be impossible, although we can get pretty close (try squaring $\frac{7}{5}$). Since there is no fraction equal to $\sqrt{2}$ it is not a rational number, which is why we call it an irrational number. Another well-known irrational number is π .

Any number, rational or irrational, has a decimal expansion. Sometimes it goes on forever. For example, the rational number $\frac{2}{11}$ has the decimal expansion 0.181818... with the 18s repeating forever. Every rational number has a decimal expansion that either stops at some point or ends up in a repeating pattern like $\frac{2}{11}$. Irrational numbers also have infinite decimal expansions, but they don't end up in a repeating pattern. From the decimal point of view we can see that rational numbers are pretty special. Most numbers are irrational, even though the numbers we use on a daily basis are more frequently rational.